CONCISE HISTORY OF LOGIC

Heinrich Scholz

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Translated by

Kurt F. Leidecker

PHILOSOPHICAL LIBRARY
New York

Translated from the German
Abriss der Geschichte der Logik.

Library of Congress Catalog Card No. 60—13660

Printed in the United States of America

PREFACE

The reader of this *Concise History of Logic* is entitled to know what the objections to this book are and why it was nevertheless published.

Carl Prantl (1820-1888) produced between 1855 and 1870 a standard work and source book for the history of logic from Aristotle to the end of the 15th century in which it is possible even now to appreciate an admirable mastery of the material, an exemplary punctiliousness in presenting the sources, and a nearly equally perfect intuitive certainty with which the material has been selected. For the history of modern logic there simply does not exist any work which could remotely be compared with Prantl's. Indeed, such a work will be written only when more shelf footage of monographs is available and each monograph can be considered on a par with the one Louis Couturat (1868-1914) wrote on the logic of Leibniz.[1]

It is, therefore, incumbent on us to state boldly that the present concise history is a hazardous enterprise. For, it is impossible to summarize knowledge which does not even exist as yet, and which cannot

be created by a *tour de force* in mere sampling of what can only be actually gotten hold of by most thorough and painstaking research, and even at that not so without reliance on one's intuition and an eye sharpened by long experience.

Another and still greater flaw in the enterprise is this. When Prantl wrote his history of logic the type of modern formal logic which is now available in the shape of symbolic logic had not yet been called into being. There was, therefore, no dependable position by which such a history could be oriented and from which it could be surveyed. For, what formal logic really is we know only because symbolic logic provided the conceptual equipment needed to answer this problem. In general, too, the extant gains registered by the modern symbolic treatment of logic have become such an essential factor in making pronouncements regarding the history of logic that we are constrained to say that an essential knowledge and mastery of the results of symbolic logic have become an indispensable condition for any and all fruitful study of the history of logic. Prantl had to rely completely on himself in sifting the material, in highlighting and playing down certain aspects. He worked under a serious handicap by virtue of the nonexistence of exact formal logic in his day. This resulted in the formation of value judgments which, measured by the standards of rigorous critical thinking now in demand, are shot through with very bad blunders. These value judgments, thus, should first be corrected. Then the entire magnificent material which Prantl spread out before us must be subjected to a fresh and thorough reinterpretation, making use of all the material contributions that have been made

since his time. However, in our endeavor we must never lose sight of the fact that the logic of antiquity, and to a considerable degree the logic of the middle ages, have come down to us in heaps of fragments.

A third and very great flaw is the multiplicity of forms in which logic manifested itself, particularly in three stages; when it was raised to the first power in the days after the *Logic of Port Royal* (1662); when it was raised to the second power after Kant; and finally when it was raised to the third power after Hegel, a stage in which we have witnessed a plethora of forms right down to the present where we are no longer able to survey them.

I have risked writing this brief history nevertheless, supported by my belief in the new logic, a belief that has aided me in conquering my inhibitions. This belief has encouraged me again and again in the difficult task of condensing the vast material into the limited space available. I owe thanks to my publisher for the understanding which prompted him to acknowledge the necessity of my going beyond the limits which I had agreed to at the outset. This made it possible to produce a little volume in which not merely beliefs could be stated, but knowledge could be spread out; knowledge, I might add, which I can back up completely by my own researches. Nothing has been referred to or touched upon in this concise history which has not passed through my fingers or which has not been thoroughly studied by me. All dates, likewise, were checked so that I have been able to correct, and that without much ado, not a few of the errors in Eisler's indispensable *Philosophen-Lexikon* as well as other, older, reference works.

I am sending this little volume into the world in

the hope that I might thereby kindle in the reader a confidence, which he might not have had before, in the new logic upon which I have based my history, hoping of course that he may overcome all obstacles with which we have to reckon. Furthermore, I possess faith that the history of logic, with the new light which can be thrown on it today, will become a beautiful and fascinating chapter of western civilization, so that at long last it may be studied with pleasure and sympathy. This accomplished, there will follow the labors of scholars as a matter of course which will close the gaps in the history of logic which we still, regretfully, have to admit today.

Heinrich Scholz

INTRODUCTION

When Heinrich Scholz wrote his *Abriss der Geschichte der Logik* in 1931 he was Professor emeritus of Mathematical Logic and Basic Research (*Grundlagenforschung*) at the University of Münster in Westphalia, Germany. He was born in 1884 in Berlin, and had already forged a distinguished career as Professor of Theology at Breslau and Professor of Philosophy at Kiel, when he became the enthusiastic champion of symbolic logic.

With penetrating insight he had written critically on war and Christianity, on the idea of Immortality as a philosophical problem, on the As-if, on Kant, on Augustine, on the concept of relativity and many other topics, some with deep cultural concern. Thus, in his *Religionsphilosophie* (1st ed., 1921; 2nd ed., 1923) he advanced the rather interesting and charitable idea that the three major religions cannot be arranged hierarchically because each represents religion in its fullness and all the ramifications of its manifestation. In 1941 he published his *Metaphysik als strenge Wissenschaft*. He died in 1958.

It was his avowed opinion that only after the

appearance of symbolic logic did it make sense to write the history of western logic. In this lies a not too thinly veiled criticism of all historians of logic, including the great Prantl. It is the point of view of the new logic, then, which determined his selection of the thinkers constituting the mainstream of formal logic—the only type of logic worth the name, according to him. His treatment of thinkers from Aristotle through the Stoics and Scholastic philosophy to Leibniz and beyond is at once unique and a marvel of condensation and critical sifting. His immense regard for Aristotle is only matched by that for Leibniz, the founder of symbolic logic, and by his contempt for all who are even mildly critical of formal logic. When he comes to men like Hegel, whom he considers a calamity, his criticism becomes charmingly skittish.

The translation of the *Abriss der Geschichte der Logik* has been made from the second unchanged edition which appeared in 1959. The limitations of space to which Scholz was committed, as he tells us in his Preface, resulted in a syntactically and grammatically highly involved style which, of course, does not present difficulties to the reader of the original who may even welcome and justify it on grounds of a higher level of precision. If we have been able to lose none of the precision while admitting our inability to reproduce the charm, we are quite satisfied. Since this history will undoubtedly remain a principal work of basic research (*Grundlagenforschung*) in logic for some time to come, a few changes in phraseology, which would have dated it too obviously, have been deemed justified, along with some corrections.

KURT F. LEIDECKER

Mary Washington College
of the University of Virginia

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ABBREVIATIONS

ADB—Allgemeine Deutsche Biographie

AMA—Abhandlungen der Münchener Akademie der Wissenschaften

APhG—Abhandlungen zur Philosophie und ihrer Geschichte, ed. by Benno Erdmann (Halle a.S., Max Niemeyer)

AT—Oeuvres de Descartes, publiées par Charles Adam & Paul Tannéry (Paris, 1897 ff.), 12 vols.

B—Eduard Bodemann, *Die Leibniz-Handschriften der Königlichen öffentlichen Bibliothek zu Hannover* (Hannover and Leipzig, 1895)

BT—Bibliotheca Teubneriana

CAG—Commentaria in Aristotelem Graeca, edita consilio et auctoritate Academiae Litterarum Regiae Borussicae (Berlin)

Couturat—Louis Couturat: *La Logique de Leibniz, d'après des documents inédits* (Paris, 1901)

CR—Corpus Reformatorum, ed. C. G. Bretschneider

Fr.—Opuscules et Fragments inédits de Leibniz, par Louis Couturat (Paris, Alcan, 1903)

KV²—Kritik der reinen Vernunft, with the pagination of the second edition of the original of 1787

Math.—Leibnizens mathematische Schriften, ed. by C. J. Gerhardt (Berlin, 1849 ff.), 7 vols.

MPL—Migne: *Patrologia Latina*

NE—Nouveaux Essais sur l'entendement humain 1704 (published in 1765)

PhB—Philosophische Bibliothek, Felix Meiner (Leipzig)

Phil.—Die philosophischen Schriften von Gottfried Wilhelm Leibniz, ed. by C. J. Gerhardt (Berlin, 1875 ff.), 7 vols.

PM—Principia Mathematica

Prantl—Carl Prantl: *Geschichte der Logik im Abendlande* in 4 vols. (Leipzig, I, 1855; II, 1861; III, 1867; IV, 1870)

SBA—Sitzungsberichte der Berliner Akademie der Wissenschaften

SHA—Sitzungsberichte der Heidelberger Akademie der Wissenschaften

SMA—Sitzungsberichte der Münchener Akademie der Wissenschaften

SWA—Sitzungsberichte der Wiener Akademie der Wissenschaften

Ueberweg-Geyer—Friedrich Ueberweg: *Grundriss der Geschichte der Philosophie* vol. II: *Die patristische und scholastische Philosophie* [11] (1928), ed. by Bernhard Geyer

WH—Wissenschaft und Hypothese (Leipzig, Teubner)

WL—Bernard Bolzano: *Wissenschaftslehre*

WW—Gottfried Wilhelm Leibniz. Sämtliche Schriften und Briefe. Ed. by the Preussischen Akademie der Wissenschaften (Darmstadt, Otto Reichl, 1923 ff.)

CONCISE HISTORY OF LOGIC

TYPES OF LOGIC

1

When speaking of logic and its history we speak of it with a certain naïveté which does not bear strict investigation. We talk about the history of logic without asking beforehand whether logic has a history at all. That such a preliminary enquiry is not an idle one we gather by taking a look at the preface to the second edition of the *Critique of Pure Reason* which appeared in 1787, in which Kant gave logic the now famous testimonial to the effect that, since Aristotle, it "has not been able to take a single step ahead and, hence, to all appearance, seems to be finished and complete." [2]

2

To every thinking person it is obvious that under no circumstances can we speak of logic in the same

sense in which we speak of the Cathedral of Cologne, or the Ninth Symphony, or the bard who composed *Faust*. For, in these three cases, as in every analogous one, we presuppose that there is one and *only one* object having the particular properties of the thing described. This can obviously *not* be maintained in the case of logic. Kant himself established an entirely new "transcendental" logic in opposition to the logic of Aristotle which he himself called formal logic for the first time. This one classical example may suffice to nullify once and for all the statement that there exists only one kind of logic, at least if we take logic in the naïve sense.

What has been established thus far? The result of our deliberation has been that we ought to gain an understanding, first of all, of the most important types of logic [3] before we are in the position of coming up with a foolproof concept of the history of logic.

3

Such an understanding may well start with Aristotle (384-322 B.C.). For it was Aristotle who thought of himself in the definite and even today unassailable sense as the creator of logic.[4] At any rate, we are indebted to him for the original form of logic, or at least, the character of its outline.

In order to project this form let us take the writings which in all probability had already been collected under the name of *"Organon"* by the Aristotelians of the first century B.C., according to Prantl. Next let us select the two courses of analytic investigation, the *Analytica posteriora* and the *Analytica priora,* concerning which we may confidently

maintain that they can be historically identified with the substance of this work.

In the *Analytica posteriora* Aristotle replies to the question of the nature of science inspired by his great master Plato. It is, therefore, visibly guided by a mathematical pattern. We may formulate his answer as follows: A science is a series of propositions, *i.e.,* a series of incontestibly true statements for which it can be maintained that they fall into two classes. To the first class belong the basic principles or axioms, *i.e.,* the remarkable propositions whose truth is so evident that they are neither capable of nor in need of proof. To the second class belong the propositions or theorems, *i.e.,* the propositions whose truth can be demontrated on the basis of the truth of the axioms.[5] Should one ask further regarding the significance of all this we could answer in Aristotle's sense: The subordination of these statements to the class of true statements can be achieved on the basis of the truth of the axioms by using absolutely correctly certain operational rules. These operational rules we designate nowadays as the rules of logic. To be sure, Aristotle himself did not designate them as such, but he formulated them in the *Analytica priora* in such a manner that they may be looked upon as the historic substance of this work.

With this step Aristotle created what Kant later called formal logic. Right here we may answer the important question which Kant himself was *not* able to answer punctiliously, that is, how far this Aristotelian logic may be designated as *formal. Apropos* we shall have to be clear in our own minds as to what we should understand by form in general and by perfect form in particular. With Aristotle we are

presupposing that we are capable of analyzing the components of any statement made—by which we understand with Aristotle an expression respecting which either truth or falsity may be asserted [6]—into two classes so as to put into the first class those components which we regard as fixed and unchanging, and into the second class those which we choose to regard as variable. These components we designate with Aristotle by letter and these we treat as variable signs, or variables in brief, *i.e.,* as signs for empty places into which something may be inserted. For the time being we shall not take into consideration what is to be inserted. And now, let us state the matter thus: By *form in general* we understand an expression in which at least one variable appears suchwise that this expression changes into a true or false statement by virtue of the fact that we substitute something for this variable or, in short, suitably change the content of this variable. By a *perfect form* we understand an expression which may be derived from a statement by substitution of *all* components considered variable by suitable variable signs. In the elementary symbolism of Aristotle the simplest example of such a form is the expression: All S are P. This is, indeed, so, for Aristotelian logic considers "all" and "are" as invariable components of a statement so that the expression just given fulfills the conditions of a perfect form.

Aristotelian logic deals with such forms and *only* with such forms. But we must not suppose that Aristotelian logic takes into its scope every imaginable form of this kind. Rather, it takes only those among them for which it is possible to formulate certain rules, the so-called rules of the syllogism. With Aris-

totle a rule of such a kind always has three perfect forms, F_1, F_2, F_3, of the sort that we can assert that every time F_1 and F_2 are true, F_3 also is true, or that by F_1 and F_2 being true, the truth of F_3 will necessarily result. For every case thus, in which we can maintain the truth of F_1 and F_2 we are also justified in maintaining the truth of F_3.

Now we can say: Aristotelian logic, or more accurately, the logic which Aristotle has established, is a formal logic in so far as it deals exclusively with forms, more strictly with perfect forms. It deals with them in such a way that it chooses among the forms those for which syllogistic rules may be formulated in the sense indicated. Of course, we do not assert that this interpretation of formal logic is anywhere to be met with in Aristotle. On the contrary, we would like to state expressly that this interpretation is to be found neither in him nor in any formal logician in the classical sense, meaning prior to symbolic logic. Let us add that the apparatus for such an interpretation was first created by Bolzano and Frege, the two greatest German formal logicians of the 19th century. We, nevertheless, stated the case the way we have because we consider justified the assertion that Aristotle himself would have formulated the matter in this or in a similar fashion had he had this apparatus at his disposal.

Two more additional remarks may be permissible. Whoever thinks a great deal about this concept of logic will see sooner or later that it is not capable of a natural expansion. For, if logic is to formulate the rules by whose application the theorems of a science may be derived from any given axioms, and a scientific system may be erected, then it does have a

large and precisely formulated task, a task whose sharp outlines any subsequent enrichment, however interesting, would only radically obscure.

Here we have touched upon the one point we would like to underscore with all the emphasis at our command. What we mean is, strictly speaking, that the very concept of formal logic—nonformal logic being, of course, the opposite concept which must be admitted as complementary—is a wrong concept. We call it wrong because the opposite concept would introduce tasks into logic which even in the most favorable case could only be ancillary to the main task. To express it differently, we would have to present things in such a way as the stylistic feeling of the ancients in general and Aristotle in particular —which still serves us well in these matters—would have never, or else, only reluctantly tolerated.

The second observation we are interposing here has to do with the structure of Aristotelian logic. To everyone who gives it a thought it must be clear that this kind of logic itself cannot be construed into a science in Aristotle's sense because it would have to axiomatize itself. Were logic called upon to do that, then it would be incomprehensible how operational rules are derived, by whose application to the given axioms it gets its own theorems. At this late date even we are unable to say to what extent Aristotle himself already was aware of that. Highly instructive in this connection is a 13th century report of Albertus Magnus transmitted to us probably via Arab sources, from which we gather that there probably were logicians in antiquity who already realized the situation and for this reason eliminated logic from the array of sci-

ences.[7] Only in this context does it become quite clear what Aristotle has done for logic by not limiting himself to simply enumerating the rules of the syllogism which he recognized as valid, but by being the first to furnish an axiomatization of logic. This accomplishment is, indeed, magnificent. Yet, one must not interpret it to mean that Aristotle thus raised logic to the status of a science, nor that he was the creator of logic. Rather, one should examine his contribution in terms of in what it precisely consists. Aristotle's real accomplishment is to have approximated logic as closely as possible to a mathematical pattern so that it received the imprint of science as he understood it. This was his great achievement, and anything more than that cannot be attributed to him.

4

At this juncture we encounter a most remarkable observation. It will dawn on us that Aristotle never used the expression "logical" in the sense in which it is current with us today. His linguistic use is best explained when we picture to ourselves a "logician" as a man who can talk well. This man could, to be sure, use syllogistic reasoning well but need not have at his disposal solid knowledge. All that is required is a supply of propositions so vague that one can infer from them *everything* with the help of a couple more or less daring *ad hoc* assumptions. For, when Aristotle speaks of "logical" proof he understands by it a proof which as such is undoubtedly correct but is, nevertheless, not thoroughgoing. It is not thoroughgoing *for the reason that* the subject of this kind of proof

operates with initial propositions which are characteristic only of the type of "logician" we have just described.[8]

Thus, a definitive name had to be coined for what Aristotle had created.[9] This, too, was accomplished by the Aristotelians, and probably by the same Aristotelians who, in the first pre-Christian century, came up with the name of *"Organon"* [10] for the logical writings of Aristotle, by which name they are still designated. From then on one was entitled to understand by logician a person who feels responsibility for this type of logic and at least understands it as much as one must in order not merely to be able to criticize it and, hiding behind some well-known names, sell it down the river to the lowest bidder.

However, we should guard against thinking of the introduction of this new term as if it had come immediately into common usage. On the contrary, it took many centuries, indeed, more than a millennium and a half, before it finally asserted itself. During all that time it was subject to considerable competition. The keenest competition came from the Stoics in so far as they designated by *dialectic* what we now call logic.[11] This term was handed down to the Latin Middle Ages by Martianus Capella (around 430 A.D.) [12] and Cassiodorus (around 500 A.D.), the pupil of Boethius.[13] It was Martianus Capella and Cassiodorus who created the textbooks which proved fundamental in medieval schools and laid at the same time the basis for the terminology of the branches of instruction in medieval education. It is, therefore, not surprising that the logic which was very famous at the beginning of the 12th century and even much later,

has come down through the ages as *Dialectica*. We mean the logic of Abélard.[14]

The expression "logic" also came to the fore during the 13th century following the tremendous upswing of logical studies due to the full availability of the logical writings of Aristotle to the Latin-oriented western world which at that time was no longer at home in the Greek language. Let us mention, for example, the much-consulted *Summa totius Logicae Aristotelis*[15] which had been passed on erroneously under Thomas Aquinas' authorship, the famous *Summulae logicales* of Petrus Hispanus (around 1250 A.D.),[16] and the *Summa totius Logicae* of William of Ockham.[17]

Then in the 16th century once more a considerable reverse set in. "Logic" was almost wholly displaced by "Dialectic." The leading logicians of that century chose for their works the title of "Dialectic." Thus Melanchthon (1497-1560) for all three editions of his logic which, let it be said, was rather modest so far as content was concerned, but was more successful humanistically: *Compendiaria dialectices ratio* 1520; *Dialectices libri quattuor* 1528; *Erotemata dialectices* 1547.[18] Thus Petrus Ramus (1515-1572).[19] Thus also the "Portuguese Aristotle" Petrus Fonseca, S.J. (1528-1599): *Institutionum dialecticarum libri octo*. These "Institutions" are an encyclopedic work whose first edition appeared in 1564 at Lisbon and afterwards was frequently republished for the next two generations.[20]

In the 17th century the picture changed all of a sudden in favor of "Logic." All logicians of note in that century decided in favor of this term. "Dialectic"

was at last eliminated from the field. Christoph Scheibler (1589-1653), the Protestant Suarez, as his admiring contemporaries called him because of his services in the rejuvenation of Aristotelianism, led the procession with a cyclopedic *Opus Logicum* (Marburg, 1633). Then came Joachim Jungius (1587-1657), so highly esteemed by Leibniz, with the *Logica Hamburgensis* (1638),[21] the German Cartesian Johannes Clauberg (1622-1655) with a thematically highly problematic *Logica vetus et nova* (Amsterdam, 1654?) which is in general chuck full of all sorts of problems of psychology and the mechanics of acquiring knowledge,[22] and somewhat later the Belgian Cartesian Arnold Geulincx (1625-1669) with his *Logica fundamentis suis, a quibus hactenus collapsa fuerat, restituta* (Leiden, 1662).[23] In the same year, 1662, there appeared the famous *Logic of Port Royal* written by Antoine Arnauld (1612-1694) and Pierre Nicole (1625-1695) under the title of *La Logique ou l'art de penser,*[24] the opus which, probably more than any other logical work of the 17th century, contributed to the adoption of the title of "Logic."

In 1670 Jakob Thomasius (1622-1684), the teacher of Leibniz, published his *Erotemata logica*[25] *pro incipientibus* (Leipzig).[26] In 1678 the famed French physicist Mariotte (?-1684) brought out his *Essai de Logique, contenant les principes des sciences et la manière de s'en servir pour faire de bons raisonnements*[27] which has been wholly forgotten yet is very much worth reading. The year 1686 saw the publication of *Institutio Logicae* (Oxford)[28] by the author of the renowned *Arithmetica infinitorum* (1656), John Wallis (1616-1703), the year 1687 the *Medicina mentis*

sive tentamen genuinae Logicae, in qua disseritur de methodo detegendi incognitas veritates (Amsterdam) [29] by Ehrenfried Walther von Tschirnhausen (1631-1708) whom we know from Leibniz' correspondence.

Against this powerful forging ahead of "Logic" even Bacon's now famous rejuvenation of the term *Organon* [30] could not prevail. We are to meet the same title twice more and prominently in the history of logic: In the 18th century with the Swiss mathematician Johann Heinrich Lambert (1728-1777) of widespread fame who wrote *Neues Organon oder Gedanken über die Erforschung und Bezeichnung des Wahren und dessen Unterscheidung von Irrtum und Schein,*[31] and in the 19th century with William Whewell (1794-1866), the meritorious author of the *History of the Inductive Sciences* who wrote *Novum Organon renovatum.*[32] Even with this he was no match for the competition. Besides, rather little of the magnificent expanse of the Aristotelian *Organon* was incorporated into these three works, least perhaps into Bacon's *Organon* which was favored by fortune hardly comprehensible and whose entire merit to a critical student may be condensed into the one catchword "induction." It must be owned that this catchword is injected into the discussion with an instinctive confidence and eloquence amounting to genius. The rest of what we read in the *Organon* others have carried to greater depth and length and at any rate far more competently.

In the meantime the term "logic" met once more serious competition in the 18th century on German soil in the persons of Christian Thomasius (1655-1728) and Christian Wolff (1679-1754), who advocated "the

doctrine of reason" or "the art of reasoning." Thomasius published in 1691 his *Einleitung zu der Vernunftlehre* and *Ausübung der Vernunftlehre*.[33] Both works are highly interesting from a humanistic point of view and are noteworthy also because they are among the first treatises on logic in the German language which were read by a wider public.[34] Christian Wolff, the renowned master of German Enlightenment, interpreted logic along with Christian Thomasius as the art of reasoning or the doctrine of reason in his small but frequently reprinted German logic entitled *Vernünftige Gedanken von den Kräften des menschlichen Verstandes und ihrem richtigen Gebrauche in Erkenntnis der Wahrheit*.[35]

In Germany the new name met with considerable and favorable reception during the 18th century. Even Wolff's critics were in favor of it. The most important one among them around the middle of the century, Christian August Crusius (1712-1775), preferred, to be sure, as title for his logic, the somewhat involved expression *Weg zur Gewissheit und Zuverlässigkeit der menschlichen Erkenntnis* (Leipzig, 1747),[36] but in the text he speaks exclusively or nearly so about the doctrine of reason. *Vernunftlehre,* as a title, finally appeared on the title page of the most popular logic of the second half of the 18th century, the work by Hermann Samuel Reimarus (1694-1768), the professor who taught at the Hamburg *Gymnasium* and whom Lessing made famous. This logic appeared at Hamburg in 1756 and came out in its fifth edition as late as 1790.

If we now ask, at last, by whom the expression "logic" was definitely domesticated in German litera-

ture, we come up against one of the most remarkable phenomena in its history. We would certainly have to mention, above all others, Hegel (1770-1831), the man who strangely enough criticized Aristotelian formal logic till there was nothing left of it. In consequence, his tremendous life's work was afflicted with a blemish which can hardly be overestimated. Because of the wide popularity of Hegel's philosophy his concept of logic even today hampers quite noticeably any serious work in logic in the Aristotelian sense; that is, logic which is subject to rigorous checks. Hegel published his tripartite speculative main work during the years 1812-1816 as *Science of Logic* and thus contributed more than anyone else toward gaining ultimate recognition for the term "logic." [37]

5

The *Logic of Port Royal* of the year 1662 was the first important work which went in principle beyond the limits of formal logic. This is not to be understood as if it neglected formal logic. On the contrary, formal logic in this work is expounded with a persuasion we cannot esteem highly enough, especially after the onslaught of Bacon's in 1620 and Descartes' criticism which must be taken even more seriously. However, there is a conscious demolition of the boundaries of logic. This is indicated in the title which was chosen consciously not to read *"La logique ou l'art de raisonner,"* but to read *La logique ou l'art de penser.*[38] With this title they meant to gain not only the required foundation for a rather detailed treatment of Descartes' new doctrine of ideas, but

above all also the groundwork for the new methodology which operates with the Cartesian *Regulae ad directionem ingenii* [39] and the far superior rules of Pascal.[40] So far as I can see, methodology is here introduced into logic for the first time explicitly.

Johann Heinrich Lambert, a century later, went in his *Neues Organon* of 1764 [41] considerably beyond the framework of logic we have thus far considered. In this work semeiotic and epistemological discussions occupy such a large space that the syllogism which is here presented perhaps for the first time as a theory concerning "the laws of thought" strikes one almost as an enclave despite its relatively detailed treatment.[42]

Thus, gradually, a new, *additional* shape of logic takes its rise, a logic which we could designate as an *expanded formal logic* in view of its relation to the sharply defined formal logic in Aristotle's sense. We should add, however, that this extension cannot be regarded as a natural development. Quite differently it must be considered a superimposition of an entirely heterogeneous material on the logic of Aristotle.

6

The same Lambert furnished us in 1771 with a two-volume *Anlage zur Architektonik oder Theorie des Einfachen und des Ersten in der philosophischen und mathematischen Erkenntnis* [43] which had no longer any connection with formal logic at all, but was much more closely related to ontology in Wolff's sense. This work may be looked upon as the first modern theory of categories. It forms the beginning of

a series whose latest and highly regarded member is Rudolf Carnap's *Der logische Aufbau der Welt* (1928). The *Kategorienlehre* by Eduard von Hartmann (1896),[44] among other works, likewise belongs to this group.

We can note, of course, only in passing, that the theory of categories became decisive in the development of an entirely new concept in logic with the absolutely original interpretation which Kant gave it. It is the famous concept of *transcendental* logic which Kant set up over against *formal* logic. We remember, of course, that Kant also gave formal logic its name. This new transcendental logic has only a highly problematic connection [45] with the "forms" of Aristotelian logic [46] which will not bear exact investigation. Not only can transcendental logic stand independently when this connection with formal logic is severed, but when disengaged it can be better appreciated for what it is.

What, then, are the Kantian categories? If we consciously leave mysticism aside, whose adumbrations make the categories occasionally murky even in a Kant, we can say at the outset that they are meant to be those concepts for which the following may be asserted in a first approximation: A given content of perception P is then, and then only, an element of "experience" if P can be subsumed under one of these concepts suchwise that by "experience" we are to understand in a first approximation that interpretation of the world of perception for which it is possible to bring about objective communication, that is, a communication which is obligatory for all subjects who are capable of making rational judgments. The first

and basic task of the new transcendental logic would be to establish and "justify" these categories. In this concise treatment we must dispense, of course, with fixing anywhere near precisely the very difficult concept of "justification."

Upon this spade work would follow very naturally and logically a second and final undertaking. We would want to know under what conditions a given content of perception is to be subsumed under one of the established categories. The famous "system" of all principles of pure reason [47] is then meant to guide us in our formulation of the rules of subsumption. Kant himself pointed out very nicely and quite precisely the contrapuntal play generated at this fateful juncture of the old formal and the new transcendental logic. For he saw very clearly and neatly that the peculiar rules of transcendental logic are rules of judgment, as it were, instructions for *particular* subsumptions in contrast to the corresponding rules of formal logic which prefer to leave the *particular* substitutions to the judgment of the practicing logician.[48] The rules of formal logic are, therefore, analogous to the replacement rules in the *dictum de omni* (which we cannot describe here in their correct formulation) and to the rules of substitution in symbolic logic.

Soon thereafter, research along the lines of this new logic coincided with a study of the great Englishmen, Locke, Berkeley and Hume, by whom Kant himself was deeply influenced. In Germany today such research is carried on in the form of "epistemology" and occasionally also in the form of "theory of science." In what follows we shall designate it as *nonformal logic*. A few remarks may be permitted at this point:

a) The over-all concept of a *"theory of science"* should be thought of as covering formal and non-formal logic and may be defined as the theoretical equipment for gathering scientific knowledge in the widest sense.

b) *Formal logic,* then, coincides with that part of the theory of science which formulates the syllogistic rules requisite for construing any particular science and furthermore provides us as a matter of course with all that may be required in a precise formulation of these rules.[49]

c) By *nonformal* logic we finally understand the entire remaining field of this theory of science, hence everything that, on the one hand, can be subsumed under this concept of a theory of science and, on the other, differs from formal logic.[50]

7

Above [51] we spoke of an *expanded* formal logic in view of the *Logic of Port Royal* and in connection with Lambert. Now we can also speak conversely, in view of Germany since Fries and with reference to England and the rest of the countries since John Stuart Mill, of a *nonformal logic undergirded by formal logic.* The first model of this logic was created by Jakob Friedrich Fries (1773-1843) in his *System der Logik* (1811).[52] He was an uncommonly keen thinker who was for an incredibly long time suppressed by the dominating influence of Hegel and the Kantian renaissance. To be sure, he put the center of gravity of his logic, without a doubt, into the nonformal field. But in his work he also dealt with formal logic so sympathetically that his treatment may be valued even

today as one of the best treatises in the nature of a mature and well-thought-through Kantianism.

Thirty years later John Stuart Mill (1806-1873) in England came out with his famous work on logic. To the outsider it presented the new logic much more clearly and brilliantly. What Mill strove for was already happily stated in the title of his main work on logic which even during his lifetime reached the unheard-of number of eight editions and today is accessible to everyone in popular editions: *A System of Logic, Ratiocinative and Inductive, Being a Connected View of the Principles of Evidence and the Methods of Scientific Investigation.*[53] Indeed, Book I ("Of Names and Propositions"), Book II ("Of Reasoning"), and Book V ("On Fallacies") of this logic in six Books are devoted exclusively to the problems of the old formal logic. Let us acknowledge that today only the theory of induction is of interest, and even that continues to exist almost entirely in secondary treatises. But there were times when Mill's work exerted considerable influence with its formal logical doctrines, but most of all with its theory of judgment, which influenced Franz Brentano and his circle.[54]

Foremost among the logic treatises of the type created by Fries and Mill there belong in German, the two best-known books on logic of the last third of the 19th century, the rambling *Logik* in three parts by Wilhelm Wundt (1832-1920) which came out in 1880-83,[55] and the more concentrated two-volume *Logik* by Christoph Sigwart (1830-1905), Vol. I, 1873, Vol. II, 1878.[56] In both works the emphasis is on methodology. We may also call attention here to Lotze's *Logik* of 1874 [57] which is stylistically quite outstanding. In this work we should stress as a matter of course the

greater tendency toward metaphysics, or to be more exact, value metaphysics, for which it may lay claim to being the first, and in a way, classical example. We say this in view of the voluminous labor of Heinrich Rickert [58] and Emil Lask [59] who are interested in value analysis and in whose writings formal logic no longer appears at all.

8

Undoubtedly the most revolutionary interpretation which the concept of logic in the Aristotelian sense received was at the hands of Hegel. Hegel's *"Wissenschaft der Logik"* [60] is linked with Aristotelian logic only by the caricature he has drawn of it in the second volume of this work. Even this caricature seems extraneous and its very presence belongs to the greatest riddles of this work which abounds in enigmas. The rest of the work is so vastly different and distant from anything that could somehow be brought into relation to Aristotelian logic that it remains inaccessible to an Aristotelian. How could an Aristotelian be expected to think himself into a "logic" which begins with the cancellation of the two fundamental propositions of the principle of contradiction and the excluded middle! For that reason alone we have to present Hegelian logic as a new type of logic. To be sure, we could have thought of incorporating it into the group of theories dealing with the categories which we discussed above; [61] but even then the work seems too unique and wayward.

9

In the year 1713 there appeared posthumously the *Ars conjectandi* of Jacob Bernoulli (1654-1705),[62] the mathematician from Basel. It is the first textbook of the calculus of probability[63] based on the ingenious ideas of Pascal, Fermat and Huyghens. In this connection we have not yet decided, and perhaps are more undecided than ever, regarding the following points. Has, with the appearance of this work, a new branch of mathematics or rather, a new logic, been created and presently developed into great significance, and has it done so on the basis of Aristotelian logic, or does this new logic, if it exists at all, embrace Aristotelian logic as a special case? The discussions of these highly interesting and exciting problems are in full swing, and we have to confine ourselves here to adducing only the most important literature for the sake of orientation.

Ranking first is the report on the Conference on the Epistemology of the Exact Sciences held at Prague in 1929.[64] The first axiomatization of the theory of probability on the basis of relative frequency was offered by the Berlin mathematician Richard von Mises: *Grundlagen der Wahrscheinlichkeitsrechnung* (1919).[65] An excellent introduction to the point of view of this author is his booklet on *Wahrscheinlichkeit, Statistik und Wahrheit* (1928)[66] written for a larger public. Also worth mentioning in this connection is the work on *Wahrscheinlichkeitsrechnung und ihre Anwendung in der Statistik und theoretischen Physik* (1931) which was meant to substantiate his

thesis on a grand scale. Even in this attempt serious mathematical difficulties became apparent. Among those who tried to remove them we must mention first and foremost Erhard Tornier: *Eine neue Grundlegung der Wahrscheinlichkeitsrechnung* in *Zeitschrift für Physik,* vol. 63 (1930), pp. 697-705.[67] The most important representative of the classical theory of probability based on the so-called equiprobability is the work of the English economist John Maynard Keynes: *A Treatise on Probability* (London, 1921).[68] We recommend for further orientation Gustave du Pasquier: *Le calcul des probabilités, son évolution mathématique et philosophique* (Paris, 1926).

10

Even at this late date we cannot assert that we have even approximately covered all possible types of logic. For in this day and age we have to expect almost anything, not only a *Logik der Philosophie* by Emil Lask (1910),[69] but also a *Logik der Aesthetik* by Franz J. Böhm (1930),[70] a *Logik des konkreten Begriffs* by E. Mannheim (1930), a *Logik der Gemeinschaft* by H. Pichler (1924), and so forth. We shall not go on with this enumeration. The works referred to in which not even an attempt has been made to interpret the term logic they have appropriated, suffice to demonstrate all too well that logic is being maneuvered into a crisis which threatens to dissolve its very form.

11

In this brief history we shall henceforth take into consideration only the first type of logic we have mentioned, formal logic. Two reasons contribute to our decision: First, the consideration of space which is limited and makes it mandatory to sift the material severely; secondly, the factor determining the principle of our selection. Formal logic is the only logic today which has been studied thoroughly enough in the form of symbolic logic to provide us with a point of vantage from which it is possible to survey the past history of logic even though we have to limit ourselves severely.

12

Thus we may be said to have saved *the* logic in the pages of this concise treatment of its history. The position of symbolic logic enables us to get an overview of this logic which permits us to talk in rather precise terms about its *history,* at the same time taking our stand against Kant.[71] Our treatment will have to show in what follows:

a) The most essential contributions that have been made to the Aristotelian core of logic.

b) The most essential changes in the nature of deformations that have been imposed on Aristotelian logic.

c) The most essential accomplishments along the lines of preserving an understanding of the significance of logic in Aristotle's sense and elucidating the

basic problems with which logic is saddled, as are the rest of the fields of philosophy. The criteria which are required in the course of the investigation will be taken from the ripest form of this type of logic, symbolic logic.

13

The history of logic thus viewed falls, therefore, into two sharply divided chapters. The first chapter will cover the classical shape of formal logic from Aristotle to the present and must garner everything *not* inspired by Leibniz' idea of symbolic logic. The ordinary division into antiquity, Middle Ages, and modern times means practically nothing for this type of logic. The second chapter will cover the shape of formal logic beginning with Leibniz, taking in everything inspired, consciously or unconsciously, by Leibniz' idea of symbolic logic.

THE CLASSICAL TYPE OF FORMAL LOGIC

A

In order to appreciate the spirit in which the one to whom we owe logic wrote and thought, one must be able to think platonically. "God created for us and endowed us with the power of vision so that we may become aware of the movements of Universal Reason in the heavens and utilize them for the regulation of the movements of our own reasoning activity. Both are related, the one with the other, in so far as it is possible at all that erratic movements can be related to true movements. These (true movements) we should impress upon our memory. Of the correct thought processes induced by nature we should appropriate as much as will enable us through imitation of the movements of the Godhead which are never deranged, to guide as far as possible the errant movements of our own inner being back into their proper path."

With this magnificent conception of the regulative activity of man's reason Plato created the high

level of thinking on which the work was conceived and from which we have selected the concept of logic in Aristotle's sense.[72]

This work is the *Organon*. Even the most condensed outline of the history of logic must contain a few sentences concerning this *Organon*. In its traditional structure it is divided into five parts arranged as follows:

1) The categories, that is, the theory of the classes of things which in a statement of the form "S is P" may appear either in the place of S or in the place of P.

2) A theory of judgment under the title of

περὶ ἑρμηνείας

(*de interpretatione*).[73] This theory of judgment contains [74] the principle of the excluded middle which is fundamental in Aristotelian logic. It contains it in the shape in which it is most accessible to a preliminary understanding of the criticism which the Dutch mathematician L. E. J. Brouwer levelled against it for many years: "Either all S are P or there is an S which is *not* P. Hidden in this formulation is the presupposition that a statement of the form "*Not* all S are P" is equivalent to a statement of the form "There is an S which is *not* P." For, at first, we only get: "Either all S are P or *not* all S are P." Now, Brouwer questions the admissibility of the transition from "*Not* all S are P" to "There is an S which is *not* P," which is assured by the Aristotelian equivalence. Likewise, of course, in the case of the transition from "*Not* all S are *not* P" to "There is an S which is P." He questions it by giving a reason which must be taken quite seriously, that the mathematician is justified in speak-

ing of the existence of an S which is a P only when he can "construe" such an S, and not right after being able to show by some sort of indirect proof, however sagacious, that *not* all S can be *not* P.[74]

3) *Analytical Investigations, First Series (Analytica priora),* in two books. They consist, first of all, of a core, the Aristotelian rules for valid syllogisms.[75] Let us merely observe that they are much richer by virtue of detailed examinations of statements regarding necessity, impossibility and possibility, than one is able to learn from scholastic logic which is limited to *"Barbara," "Celarent,"* etc. These books contain a considerable amount of valuable additions to this core[76] among which four items should be highlighted:

a) The chapter on the *oblique figures*[77] which are quite inconvenient to Aristotelian syllogistics. They are the figures in which at least one term does not (merely) appear in the nominative case, but (also) in another case. They are best explained by an example.

The square of an even number is itself an even number.
Six is an even number.
The square of six is an even number.

b) The very interesting chapter on the interpretation of *negation.*[78] In this chapter negation is interpreted in such a way that, *e.g.,* the expression "Ten is *not* a malicious number" passes over into a true statement. Today we do not look upon such and far more "innocuous" expressions in logic as statements

but as meaningless verbal compositions. In the present case we adduce as reason for the meaninglessness of the expression the fact that the maliciousness of a number has never been defined.

c) The four basic chapters on the *relations* existing between premises and *conclusio* in a syllogism.[79] It is stated here for the first time that from true premises only true inferences can be drawn, from false premises true and false inferences, that false propositions may be inferred from premises of which at least one is false, while true propositions may be inferred from premises no matter whether true or false.

d) The chapter on *induction*.[80] This chapter, we admit, must be counted among the few writings which almost make no longer any sense to us. There is a chance that tradition garbled the passages already at an early date. But this criticism does not touch the key word "induction," and it must at all events be pointed out that this word is the Aristotelian germinal cell of a much later "inductive" logic.

4) *Analytical Investigations, Second Series (Analytica posteriora)* in two books.[81] In the case of this work also we may speak of a core and rich deposits. The core is essentially identical with the content of the first book, *i.e.*, the theory of the conditions which the initial propositions (axioms) of a science must satisfy in the Aristotelian sense. The second book furnishes the additions to this core. The highlights of its contents are particularly the theory of definitions, the characteristic division of the definitions into nominal ones and essential ones[82] and a remarkable chapter on how to find definitions.[83] The theory of definitions with the characteristic statement regarding

the indemonstrability of definitions [84] was written by Aristotle as a sort of appendix to his axiomatic.

5) *Topical Investigations (Topica)* in eight books, with a ninth book on the fallacies *(De sophisticis elenchis)*. They were written for beginners in dialectics, Aristotelian style. These investigations are called topical on account of the *topoi* contained in them. But what is a *topos?* Aristotle owes us the answer to this question. Hence we have to orient ourselves by the examples which we meet throughout this work. Then we could say the following: If we understand with Aristotle by dialectic the art of proving or disproving a given plausible proposition with the help of certain plausible premises; if, furthermore, we understand with Aristotle by a dialectician a person who masters this dialectic, then one may interpret the Aristotelian *topoi* as those very general propositions (maxims) which may be recommended to beginning dialecticians as guiding principles, the reason being that the knowledge and application of these propositions materially facilitate proving and exercising control.[85]

The classification of the *topoi* is essentially determined by the key words *accidens* (a property which attaches to an individual of a class only occasionally and in this sense "accidentally"), *genus* (property which attaches to all individuals of one class, but not necessarily to these alone), *proprium* (property which attaches to all individuals of one class and to these only), *definiens*. We know that these key words were later expanded to the *quinque voces (genus, species, differentia, proprium, accidens)* by Porphyrius

(around 275 A.D.) and Boethius (around 500 A.D.). Aristotle had introduced these terms for a purpose and with a penetrating insight admirable even today, yes, especially today. But detached from that purpose they attained a disproportionate fame during the Middle Ages.

Here again we have talked about the core only of this work (Books II-VII), and again we have to mention subsequent enrichments. To these belong the Introduction (Book I), the nicely concluding reflections of the eighth book, and the ninth book on fallacies which bears witness to a supreme command of the technique and "theory" of the syllogism.

Now the following remarks are in order:

1) Aristotle's *Organon* is generally speaking still the most esthetic and instructive introduction to logic written by any man.

2) Today there is only one dependable orientation for anyone not knowing Greek. It is the English translation which appeared at the Clarendon Press at Oxford under the direction of the highly deserving Aristotle-scholar W. D. Ross.[86]

3) In order to derive maximum benefit from one's study one should run through the parts of the *Organon* in reverse order.[87] For they originated in all likelihood in this sequence, far differently from the way in which a textbook would have been projected. Aristotle did *not* write a textbook of logic (no more than he wrote a textbook of metaphysics), and he never thought of compiling such a textbook. Even his most mature reflections are still marvellously *in*

statu nascendi so that they can be compared to nothing else nor be substituted by anything else.[88]

4) Because the "categories" were placed at the head they were emphasized disproportionately by posterity. It is, therefore, doubly necessary to point out the difference in levels between them and the rest of the parts of the *Organon.* This difference is so great that the genuineness of the work may well be called in question. It is remarkably thin and treats things so positively as if unaware of the problems. Thus it is quite different from the peculiar flexibility of thought which permeates the remaining parts of the *Organon.*

5) The summary to which we have had to confine ourselves, suggestive though it may have been, may, nevertheless, have given an inkling of the superabundant wealth of themes that have been touched upon in the *Organon.* With this in mind we must add, however, that *until now we have not met a type of logic worth mentioning for which there could not be found some point of contact in the Organon.* All the more admirable is the genius of Aristotle who was able to intercalate the first sketch of formal logic into his work. He did it in such a manner that the history of this enclave has become one of the most interesting chapters of the whole history of philosophy in the western world, yes, one of the most monumental ones at that, if we keep in mind the advance it was capable of achieving after Leibniz developed its possibilities.

6) The decisive step beyond Plato by which Aristotle became the creator of formal logic, was his re-

flection that the renowned platonic disjunctions which operated quite smoothly in practice with the principle of the excluded middle, were not capable as yet of furnishing a single syllogism. For, supposing I know only that S, in the sense of a complete disjunction, is either P or Q, I cannot conclude anything as yet from this fact. I can come to a conclusion only if I know further that S is not P, or not Q. Aristotle saw that, and the decisive step by which he did become the creator of formal logic was formulated fully and clearly in the *Analytica priora* I, 31. It remains for us to be amazed that the Aristotelian logic does not begin with the disjunctive syllogism, indeed, does not deal at all explicitly with these inferences, but that their thorough treatment was left to the Stoics.

7) Aristotle was not quite so successful when it came to describing in abstract terms the method of inference. What we are really doing when we infer in Aristotle's sense must be learned from Aristotle's syllogistic rules and their numerous exemplifications. We cannot get it from the famous definition [89] of the syllogism:

> ἔστι δὴ συλλογισμὸς λόγος ἐν ᾧ τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει διὰ τῶν κειμένων.

For inferring in Aristotle's sense is *not* a "discourse in which certain assertions are made and something different from what has been asserted occurs necessarily on the basis of what has been asserted," but it is an activity, an operation or process which presupposes a certain disposition of elements and is composed of certain subsidiary activities. The presupposed disposition comprises three statements which differ from

each other. They are p, q and r, which possess common terms which in turn must be described more fully and, moreover, must satisfy certain conditions which likewise have to be defined more fully. A valid syllogism in Aristotle's sense always presupposes such a disposition of elements. And in what does such a syllogism consist? It consists of two subsidiary operations: i) Stating the truth of p and q, and ii) incorporating r into the class of true statements.

Apparently this is something radically different from what Aristotle, and with him the Aristotelian school logic down to the present, have furnished us with in the way of a description of the syllogism. Even *with* the correction we cannot be said to have understood to any degree syllogistic reasoning as we are constantly practicing it. For we still would have to omit at least the syllogisms containing only one premise and treat them as "conclusions" by emphasizing disproportionately some unessential characteristics.

And yet, who among Aristotelians has corrected the master on this point? Let someone point him out to us and we shall gladly prepare him a place of honor in the history of logic which he would fully deserve! At least Aristotle interpreted the syllogism in such a way that we are not obliged to take it as a *judgment.* In order to do justice to the instinctive accuracy with which Aristotle chose the colorless expression *"logos"* for characterizing the syllogism, let us remember that even the great Bolzano [90] thought of syllogisms as a class of judgments.[91]

Above all we are constrained to say that in the Aristotelian logic of the schools, the doctrine of the

syllogistic "principle" has played a considerable rôle for centuries. It is one of the dimmest chapters of this type of logic.[92] The older logicians identify this "principle" with the totally misunderstood Aristotelian *dictum de omni et nullo*.[93] Since Kant [94] logicians more recently prefer in general the *"nota notae est nota rei"* which they lift by a *tour de force* from *Cat. 3*, p. 1 b, 10 ff., and present as a formulation of this "principle." Aristotle is not responsible for any of these principles. You will not even find a trace of these shady machinations in his work. This may be owing to the fact that the Aristotelian syllogistic presupposes *much more* than a single principle, to wit, presupposes apart from *"Barbara"* and *"Celarent"* all the operational rules requisite for the reduction of the rest of the 12 Aristotelian *modi* to this rootstock.

8) With a feeling of superiority borrowed from Kant one is in the habit of referring to Aristotelian logic, if it is to be deemed worth considering at all, as an "elementary" logic. Such a reference should either not be taken seriously or be rejected as false. For, if taken seriously, it is, in comparison with what we see around us today, just as false as the rubricization of Euclidian geometry under the concept of "elementary" mathematics. Today we consider arithmetic in the pregnant sense, that is, the theory of natural numbers, the "elementary" mathematics.[95] All the rest is *"non*elementary." Euclidian geometry is not "elementary" at all, since in an analytic interpretation, such as was made available by Descartes for calculations, it presupposes the complicated theory of real numbers. Aristotelian logic bears exactly the same relation to "elementary" logic in the present-day

sense. Modern "elementary" logic is a logic of *statements, i.e.,* a logic in which only "forms," more precisely perfect forms,[96] occur which have the property that the only variables appearing in these forms are so-called variables of statements, that is to say, symbols for zero places into which nothing else but statements may be inserted. Aristotelian logic, on the contrary is, depending on the case, a logic of *predicates* or *concepts,* or a logic of *classes.* For in Aristotelian logic only "forms" occur, such as "All S are P." Apparently we could also say instead that "The predicate or the concept of being-S implies the predicate or the concept of being-P," or "The S-class is contained in the P-class." Therefore, interpreted in the discriminating judgment we expect of a logician nowadays, Aristotelian logic is a "*non*elementary" logic.

B

Now, it is the chief merit of the Stoics to have invented this elementary logic or at least laid the groundwork for it. Of course, this merit is not minimized by the fact that it is being recognized only now.[97] Furthermore, it is not to be abridged because the Stoics themselves had no knowledge of the basic significance of their logic of statements, to judge by the fragments which are all that is left to us by fate, a fate resting heavily on the history of logic.[98] Greater importance must be assigned the precursors of the developed Stoical theory of hypothetical and disjunctive syllogisms.[99] They are Theophrastus (about 371 to about 288 B.C.) and Eudemus (around 320 B.C.), the pupils of Aristotle who attained to

significance in the history of logic. Indeed, it was they who were the first to enhance Aristotelian logic by these syllogisms [100] and thus laid the foundation for the logic of statements. For if, in agreement with tradition, we formulate merely the *modus ponens* in the sense of Theophrastus and Eudemus, we get [101]

If p, then q
Now, p holds true
Therefore q also is valid.

We, thus, arrive at an expression in which only variables of statements appear as variables. We should never forget this when we think of these two men. Likewise we should also remember always the merit they have earned in expanding the logic of predicates and classes. This they did by introducing the five syllogistic rules in which they followed purely formal reasoning which in our view is the more valuable. The later so-called fourth or Galenian figure corresponds to these five rules.[102]

However, these two logicians, highly respected as they are in their places, do not detract from the accomplishment of the Stoics. For it was the Stoics who construed the logic of statements for the first time strictly along synthetic lines. We suspect that all that came later was essentially the work of Chrysippus (about 280 to about 205 B.C.):

εἰ μὴ γὰρ ἦν Χρύσιππος, οὐκ ἂν ἦν Στοά.

The acknowledgment did not fail to materialize. Grateful for his accomplishments modern historical writers have magnanimously bestowed on him the obscure honorary title of "first scholastic."

The foundation of Stoical logic is, as has been

said, the statement—in Aristotle's sense, a thing which is either true or false. Even the subsequent Aristotelian concession to the metaphysics of freedom must be expressly set aside here, the concession which he made in the highly strange and difficult but interesting ninth chapter of the theory of judgment whose history remains to be written. In the passage alluded to, Aristotle singled out the statements about future contingent events, *i.e.,* events which are conditioned by the exercise of the "freedom" of the will, from the class of statements. The reason, highly peculiar and unconvincing, he gave for singling out these statements is that the truth of such statements would imply the necessity, the falsity, however, the impossibility of the event in question. Among other things, in Cicero's essay *de fato* there is preserved a highly interesting report to the effect that Epicurus (342/1-271/0 B.C.) made much of this *aperçu* and in his metaphysics of chance maintained that presumably *all* statements with regard to the future are neither true nor false. By virtue of this his declaration of war against the excluded middle, which we might call his brand of Protestantism, Epicurus managed to live on in the history of logic.[103] It is the only thing he has left behind in logic and it remains for us to warn each and all in heaven's name not to make him a precursor of Brouwer.[104] For it never occurred to Brouwer to declare the principle of the excluded middle as *false.* He likewise never maintained that there are statements which are neither true nor false. All he maintained was that it is not permissible to apply this principle in a *bona fide* mathematical proof. In other words, he called for a logic into which this principle is not incorporated at all; but he never had the least

intention of requiring logic to operate *denying* the excluded middle. Above all he differs essentially from Hegel whose logic asserts for *every* statement its being neither true nor false.

Thus it was the Stoics who with radical determination demanded that every statement be either true or false. They eliminated the Aristotelian restriction (though formally acknowledging Aristotle's reasoning) by declaring themselves sharply against Aristotle's metaphysics of freedom and in favor of an uncompromising ontological determinism.[105] Having done so they proceeded with their reconstruction systematically. First came the "simple" statements, that is, statements whose components are not themselves again statements. Next came the "nonsimple" or "composite" statements.[106] Here we find among other things an exact characterization of the if-so assertions in the following table of values [107] which interests us greatly but which Prantl declares excessively stupid:

p	q	If p, then q
true	true	true
false	true	true
true	false	false
false	false	true

What is the meaning of this? It means that a statement of the form "If p, then q" is false and then false only when p is true and q is false. It took more than two millennia for this nice discovery to be made a second time, and for it, in a truly ingenious manner, to be utilized in the simplest imaginable construction of an exact logic of statements by a natural extension of this method to *all* composite statements. We shall

write this to the credit of the Stoics, particularly Chrysippus, the "first scholastic." We can see as a matter of course that with such a foundation, the hypothetical and disjunctive syllogisms upon which much care and devotion have recently been spent,[108] also attain a new and sharper profile.[109]

The Stoics started with the proposition that mastery of logic is a humanistic necessity, *not* a mere sufficiency, and they did so at a time when this was *no longer* considered a matter of course, as in Epicurus' time.[110] We owe them the acknowledgment that their charming belief was followed by just as charming deeds.

C

For many centuries thereafter there appeared nothing that could be compared with the accomplishments of the Stoics. Even the little that did come out can only be touched upon here. In the second century A.D. we meet two figures: First, Alexander of Aphrodisias, a professor of peripatetic philosophy at Athens, (198-211 A.D.), the classical interpreter of Aristotelian logic whose work is inestimable. He brought out a very considerable reproduction of Aristotle's theory of the syllogism which also contained original material. His fundamental logical ethos could kindle anyone's enthusiasm even today.[111] Then there was Galen (129 to about 200 A.D.), the famous physician. The reason for our mentioning him here is not because of the Galenian syllogistic figure which has been attributed to him, probably wrongly, but because of the magnificent conception of a *Logica ordine Geometrico demonstrata.*[112] For us Galen was

the first who called for a strict axiomatization of logic and hence postulated within the limits of possibility [113] what was later for the first and only time fulfilled for classical formal logic by the *Logica demonstrativa*. This logic was written by Girolamo Saccheri (1667-1733 A.D.)[114] whose fame in history rests on the parallel axiom and the prehistory of non-Euclidean geometry. His book, which is almost totally forgotten but cannot be esteemed highly enough in the context in which we are mentioning it, is extant in Germany only in a single copy in the University library at Münster i. W.

What the Latin Middle Ages owe to the logical writings of Boethius (about 480-525 A.D.) should be looked up in paragraph 14 of *Ueberweg-Geyer*. They should be treasured as cultural accomplishments of world-historical scope. In *Ueberweg-Geyer* may also be consulted the very important distinction between genuine and attributed writings which is necessary for a proper evaluation. Boethius is, above all, to be taken into consideration in the history of logic in our sense for his thorough treatment of hypothetical syllogisms.[115] Concerning the history of the *"Logica ancilla theologiae"* which has not yet been written up, we call attention to the principle of excluded contradiction by Peter Damiani.[116] So far as I can see it is the first straightforward attempt to make Aristotelian logic explicitly unavailable for theology. This antilogical prince of the Church was opposed, around 1250, by that Petrus Hispanus who was able toward the end of his life to exchange the chair of logic for the *cathedri Petri*. His famous *Summulae logicales* [117] —which, by the way, are *not,* as Prantl maintained, dependent on Byzantine logic, according to recent

researches, but belong entirely to the western world [118] —with their theories of the *propositiones exponibiles* and *de terminorum proprietatibus* provided the foundation for reflections whose importance has been confirmed to a considerable extent by modern exact logic. With this is to be compared the *Philosophische Grammatik* which Bolzano included in his splendid *Logik* [119] and the 16th and 17th paragraphs of Bertrand Russell's *Introduction to Mathematical Philosophy* (London 1919).[120]

Peter Ramus, of doubtful fame because of his sansculottic Aristotle critique, had more luck than sense when he supplemented the three syllogistic figures of Aristotle by two new *modi,* each around the middle of the 16th century.[121] These *modi* arise when we interpret the middle term in the proper Aristotelian figures as *terminus discretus sive singularis.* By this we mean, more or less accurately, expressions which modern exact logic typifies as characterizations of individuals or expressions of the form "the So-and-so." An example would be the author of the *Critique of Pure Reason.* By admitting such terms we get, for example, the following complement to *"Barbara":*

The creator of the infinitesimal calculus was a mathematical genius.
Leibniz was the creator of the infinitesimal calculus.
Leibniz was a mathematical genius.

Today we know that such syllogisms are, indeed, autonomous with respect to those of Aristotle. For they demand an entirely new and not at all simple apparatus for their demonstration. Thus we must give

the *sex modi Ramistarum* the respect that is due them.[122] We should add, however, that in the discussion of the *Syllogismi expositorii* by Duns Scotus (around 1300),[123] William of Ockham (around 1330),[124] and Melanchthon (*Erotemata Dialectices,* 1547),[125] the way had already been paved for these syllogisms of Ramus. Furthermore it must be said that Ramus was far from formulating explicitly the rules belonging to these syllogisms. What would Aristotle have said to this sort of modesty in logic?!

The irruption of Ramism into Aristotelian logic had a very interesting sequel in the 17th century. No less a man than John Wallis, the English mathematician who was a central figure in the promotion of the theory of infinite series, subjected in the chapter *de modis propriis* of his *Institutio logica* III, 10 (Oxford, 1686),[126] Ramus' additions to Aristotelian logic to keen criticism. These contributions of Ramus' must have been discussed a great deal in England. The result was that they all could be brought to disappear by a thoroughgoing formal identification of the singular statements with the universal ones. Wallis himself vigorously carried out this identification, first in a thesis published for the first time in 1643, and later reprinted in the *Institutio:*[127] *propositio singularis, in dispositione syllogistica, semper habet vim universalis.* As reason he gave the fact that in a singular statement the predicate term in question is predicated in its entire scope by the subject term in question.[128] This is, however, not supported by a keener feeling for logical style. Wallis seems to have been the first to carry through strictly with this "interpretation." Later it was expounded with identical justification but without reference to Wallis and the *Syllogismi Ramis-*

tarum in the *Logic of Port Royal.*[129] It was finally pressed home by the great Leonhard Euler (1707-1783) in the year 1761 [130] with such force that until the advent of symbolic logic doubts could no longer be heard as to the sensibleness of this "interpretation."

Among the writings of the 16th century the *Opera logica* of Jakob Zabarella (1532-1589) must be rescued from an undeserved oblivion. The first edition appeared probably in 1578, the *editio postrema* in 1623 at Frankfurt. In my opinion, and speaking with reference to the interpretation of Aristotelian logic, these *Opera* are of considerably higher quality than the *Logische Untersuchungen* in two volumes (1840) [131] of Adolf Trendelenburg (1802-1872), the well-known and meritorious Aristotelian of the 19th century, which many even today esteem highly.

Among the productions of the 17th century the *Logic of Port Royal* [132] is still quite readable. One could envy the French for possessing a logic written in their own language which can be compared to any work of another nation so far as easy readability is concerned. This is of considerable merit if we are reminded that the work was compiled by two followers of Descartes, two men, that is to say, who had grown up under the most radical Cartesian criticism of this logic and were really carrying other things in their head. To be sure, this logic is *not* profound. With the exception of the *Logica Hamburgensis,* it lacks profundity as do the rest of the logics of the 17th century including the *Medicina mentis* of Tschirnhausen [133] which holds one's interest only where, without mentioning Leibniz, it operates with Leibnizian ideas. And where the *Logic of Port Royal*

offers something original, as in the theory of the scope and content of concepts [134] which appears here for the first time, it has done more harm than good.

Incomparably more profound, and in general the most significant logic of the 17th century, is the *Logica Hamburgensis* of Joachim Jungius (1587-1657).[135] Leibniz had such a high opinion of it that he repeatedly placed the author on a level with Galilei and Kepler.[136] As a matter of fact it was written with the rare mind of a man who is incapable of committing logical dilettantisms. Substantially the highest accomplishments of the *Logica Hamburgensis* are the following contributions to the theory of the syllogism:

1) The introduction of the *"aequipollentiae per inversionem relationis."* [137] Example: David is the father of Solomon; therefore Solomon is David's son. And *vice versa.*

2) The introduction of the *"consequentiae a compositis ad divisa"* and *"a divisis ad composita"* [138] which even Aristotle [139] was not able to tackle fully. The reason is the same as would even nowadays make an indomitable follower of the "natural," nonsymbolic logic sit up and take notice. Here are examples: 12 is divisible by 4 and by 3; therefore divisible by 4. —12 is divisible by 4. 12 is divisible by 3. Therefore 12 is divisible by 4 and by 3.

3) The introduction of the *consequentiae simplices a rectis ad obliqua procedentes, i.e.,* deductions which pass from statements with terms in the nominative to statements in which at least one of these terms appears in a different case.[140] The most famous example is the following because it was first specially selected

by Leibniz and then given a correct formal expression not until the arrival of symbolic logic: *Omnis circulus est figura. Ergo quicunque circulum describit, figuram describit.*

4) The penetrating discussion of and improvement of the theory of "oblique syllogisms." [141]

And now we pass from Jungius immediately to Bolzano, a jump of two centuries, for Leibniz and his followers do not belong in this context. Moreover, for the entire 18th century nothing basic and essential may be added to our notice of Crusius and Lambert,[142] and especially, of course, Saccheri.[143] Perhaps we should make the observation in the case of Crusius that, indirectly, he earned recognition in logic mainly because of his struggle against the deducibility of the principle of sufficient reason from the axiom of the excluded contradiction (maintained by Christian Wolff). The result was that he liberated logic from a "procedure of proof" which belonged to the worst things which occurred in its history since Aristotle.[144] To be sure, Crusius did this with insufficient means and, in a sense, insufficiently. At most we should mention again the logic of Reimarus [145] in so far as it is, in the five editions through which it went, an interesting example of what the philosophic public of the second half of the 18th century *no longer* expected of a good logic. Note that this was 150 years after the first appearance of the *Logica Hamburgensis* which had meanwhile been completely forgotten. The simple circles of Euler [146] still claim a place of honor because of their contribution to the popularization of Aristotelian syllogistics [147] which con-

tinues down to the present. However, Wolff's voluminous and influential logic, the *Philosophia rationalis sive Logica, methodo scientifica* [148] *pertractata* (Frankfurt and Leipzig, 1728), which appeared in many editions, ought really only be mentioned because of its curious obtuseness in most critical points and because it was essentially responsible for the table of judgments which Kant used in support of his table of categories but which by itself does not bear close scrutiny at all. Also Kant's own manifesto *Von der falschen Spitzfindigkeit der vier syllogistischen Figuren* (1762) [149] is of interest today only because nothing at all of what Kant said in this little book can be salvaged. Matters lie essentially differently in this respect with Herbart and his school. What Herbart (1776-1841) said regarding logic in the few paragraphs [150] of his *Lehrbuch zur Einleitung in die Philosophie* (1813) [151] is even today very much worthwhile and should engage our thinking. The logic of his most important pupil, M. W. Drobisch (1802-1896), under the title of *Neue Darstellung der Logik nach ihren einfachsten Verhältnissen, mit Rücksicht auf Mathematik und Naturwissenschaft* (1836) [152] we can still refer to with respect, to which it is entitled without a doubt.

Yet all accomplishments along these lines were outshone by and cannot come up to the work of Bolzano. For, in the three first volumes of his four-volume *Wissenschaftslehre* of 1837,[153] Bernard Bolzano (1781-1848) created an introduction to logic from which we can learn so much that we can only say: Take it and read! We say "introduction to logic" despite its tremendous scope in order to indicate that

the center of importance of this work is not to be looked for in syllogistic theory as such, but in the unusually keen, instructive and detailed observations which guide the reader more penetratingly and charmingly than anything *not* in line with the great Leibnizian tradition (in the sense of the logical calculus) into the problems of this type of logic. Bolzano elevated logic to such a height that the demands of syllogistics could really only be satisfied by symbolic logic. Although Bolzano built his logic on the Augustinian doctrine of truths and ideas as such, his illuminating treatment is, nevertheless, to a very large extent independent of it. These are, by the way, things which ought to be investigated more closely. Let us limit ourselves to one example.

Modern logic interprets syllogisms as deduction of judgments from other judgments. Obviously, this interpretation is meaningless so long as we do not know what is meant by deducing one judgment from another. Bolzano did find the relevant interpretation which, it must be owned, *also* does not satisfy us all around but is, nevertheless, epoch-making solely because in pursuing his objective Bolzano turned away from statements and returned to the "forms." [154] These "forms" now appear for the first time explicitly in formal logic so that with their aid Bolzano was able to obtain the most interesting interpretations not only for the derivation but also for the rest of the logically basic relations of compatibility, incompatibility, etc.[155] His charming *Philosophische Grammatik* [156] we have already mentioned. A luminous chapter all by itself contains magnificent discussions of earlier treatments of every topic of logic with special reference to Aristotle and Kant.[157] In these discussions

there is invaluable material for any *critical history of logic.*

With such illogicality did things happen in the history of logic which we are pursuing here that this great, born logician fell prey to a fate which beats the fate of Joachim Jungius. For the latter at least was read, and read by a Leibniz; but that cannot even be said of Bolzano. Hence we cannot even maintain in his case that he was forgotten. All the greater is the merit of Edmund Husserl who discovered Bolzano, a merit probably greater than that attaching to his *Logische Untersuchungen* (vol. I, 1900; vol. II, 1 and 2, 1901).[158] The considerable influence which these logical investigations have had are to be looked for in different quarters than those of the logic we are concerned with here.

Now it is possible to maintain about Bolzano that he has given us dispensation for much that happened on the historic scene *after* him, and that happened with much greater effect on history itself. We are thinking primarily of the one great work of formal logic which appeared in German during the 19th century in the non-Leibnizian tradition: the *Logik* of Benno Erdmann (1851-1921) which came out in 1892.[159] For, without gainsay, this work is the result of exemplary diligence and shows, without a doubt, a dedication to the subject under consideration which may well set the pattern for future work. Moreover, *in the way it is presented* the subject is, we might say, more than brittle, being treated on top of it very psychologically, which certainly does not accrue to the benefit of logic. Nevertheless, only those who have never read a line of Bolzano and consequently have not had the opportunity to form an adequate concept

of what a classical logician should be, will designate Erdmann's work as classical, as did the reverent editor of its third edition.

Much more deserving of such distinction is the masterwork of John Neville Keynes, the father of the well-known economist. For his *Studies and Exercises in Formal Logic* (London, 1884) [160] is the most perfect presentation of classical formal logic in general and has been equally great and beneficent in its influence within Anglo-Saxon civilization.[161]

The only other original figure among formal logicians of the 19th century after Bolzano and before Husserl was Franz Brentano (1838-1917).[162] His interpretation of the elementary "forms" of Aristotelian logic ("All S are P," etc.) and the reformation of the Aristotelian syllogism deduced therefrom,[163] constitute in the judgment of those who are well oriented in modern exact logic the most interesting corrections upon Aristotelian logic, many eccentricities in detail notwithstanding. So far as we can humanly judge, this adjustment in Aristotelian logic is of enduring character. It is, however, the only portion of Brentano's *Logik* about which this can be asserted.

The voluminous *Logik,* so rich in ideas, which the Freiburg physiologist Johannes von Kries wrote, brings us with its publication date of 1916 to the 20th century. In spite of its subtitle, *Grundzüge einer kritischen und formalen Urteilslehre,* it seems so strongly oriented epistemologically that we refer to it in this history only with reservations. However, we are obliged to report that a cyclopedic *Logik* by Alois Höfler appeared in 1922, in second edition with contributions by Ernst Mally, whose study can be recommended but only to those who can find the time for

it. The most interesting aspect of this work is the connection it has with Meinong, being essentially a logic in Meinong's sense.[164] But the *Logic* by the Englishman W. E. Johnson, whose first part appeared at Cambridge in 1921, must be considered formidable.[165] Alexander Pfänder treated logic informatively in 1921 [166] from a moderately phenomenological point of view.

Let us conclude this report with a reference to a final work which Edmund Husserl published in 1929: *Formale und transzendentale Logik. Versuch einer Kritik der logischen Vernunft.* A first reading conveys the impression that the work is to be characterized as a treatise meant to lay the foundation for a future theory of all possible types of logic.

THE MODERN TYPE OF FORMAL LOGIC

A

Mentioning the name of Leibniz is like referring to a sun rising. With him a "new life" began for Aristotelian logic whose most beautiful manifestation nowadays is modern exact logic in the form of symbolic logic. If we cannot think highly enough of the ancients, neither could Leibniz. Yet, with all his regard for Aristotle, and it was a high regard indeed, he was the man who could see far beyond the ancients, so far that something essentially new appeared to his vision.

And what was this novel thing? It was the magnificent idea of mathematizing logic. But is this idea really so new? Have we not met it already in Galen? No, not even a Galen conceived of such a logic. The reason was that the whole notion presupposes a type of mathematics which did not exist at all in antiquity. It is modern symbolized mathematics, the kind of mathematics which took its rise in Vieta and Descartes, the mathematics in the advancement of which Leibniz

earned the very highest merit by inventing the infinitesimal calculus. Galen's postulates were fulfilled by Saccheri.[167] But Leibniz' ideas are so far above those of Saccheri that in order to reach them one has to brace oneself for a mighty climb from Saccheri's work.

From the very start Leibniz looked upon codified logic quite differently from his great forerunner Descartes. For Descartes it was an annoying shackle which had to be gotten rid of if one really wanted to further science and not merely parade in questionable Sunday toggery the knowledge that had been with us for a long time past. He spoke no more kindly of it than did Brouwer, with the result that soon it became *bon ton* in philosophy to cast mud on codified logic.

From Leibniz' pen we possess a magnificent document in German to correct this unbridled tendency. It is his letter to Gabriel Wagner written at the time of his intellectual maturity. Wagner was but a prattler who owes the memory of his inglorious name to the undeserved fortune that toward the end of the year 1696 [168] brought him this letter. Even in the concisest outline of a history of logic a few passages from this letter should not be missing. "I owe I must confess that all the logical treatises thus far are hardly a reflection of what I would like them to be. And though I am looking afar off I am nevertheless obliged to confess in deference to truth and to do justice to whom justice is due, that I find much that is good and useful even in the traditional logic." [169] Then, with reference to *"Barbara," "Celarent,"* etc. he wrote: "This part is generally believed to be the most useless, but I found it differently. Even though M. Arnauld himself expresses the opinion in his *l'art de penser* that it would not be easy for anyone to go

wrong so far as form is concerned, but solely so far as the material content is concerned,[170] things are in effect quite different. Even Herr Huyghens shared my observation that very commonly the mathematical errors themselves, the so-called *paralogismos,* arise when the form has deteriorated. It certainly is not a small matter for Aristotle to have cast these forms into infallible laws and therefore become *the first,* in fact, who *wrote mathematically outside mathematics.*" [171] "Be it as it may, whether and to what extent it may be said: *purus logicus est asinus.* Scaliger wanted to say something of the sort about *mathematics;* even a teamster with no intelligence would, as soon as he leaves his wagon or his stable, not make a good servant." [172] "All that intelligence has discovered was discovered by virtue of the true rules of logic, although in the beginning such rules were not expressly noted down or compiled." [173] "In all infallible sciences, if strictly demonstrated, we find as it were, higher forms incorporated which partly flow from Aristotelian thinking, and partly avail themselves of something else yet. . . . It is as if one accepts small coins at a throw but prefers to count big coins, especially pieces of gold, and if one were to calculate diamonds one would gladly take pains to count them on the fingers of one's hand, which though assuredly the worst, is yet the most reliable calculation. However, the higher, more artful and faster the calculation, the easier it is also to miscalculate. And so it is with logic. In important, especially theological, controversial matters such as God's nature and will as would also be of concern to our souls, we would do well to take everything apart with much diligence and reduce it to the simplest possible and most tangible syllogisms, in

which case even the dullest pupil can see without a mistake what follows or does not follow. We shall also find that often in important conversation one is stuck and has to stop arguing because one has departed from the form just as one can turn a ball of thread into a Gordian knot by unwinding it improperly." [174]

This is the background with which we must be acquainted in order to comprehend the ultimate philosophical motive for Leibniz's new conception of logic. Leibniz saw that the old logic was not sufficient for a metaphysics which can place itself side-by-side with mathematics as a strict science. It was, therefore, a question of creating a new logic which would accomplish what was expected of it.[175]

And what is it that we expect of such a logic? We require of it that it will render syllogizing just as independent of thinking or the meaning content of the propositions involved in the syllogism, as modern mathematics has made calculating in the widest sense of the word right down to the magnificent feats of the modern infinitesimal calculus independent of thinking of the meaning content of the symbols involved in the calculation.

With the eye of genius Leibniz saw that the unparalleled advance of modern mathematics rests upon this unburdening of thought. Relieving thought in this way tremendously facilitates reasoning. Syllogizing is thus freed of all sorts of unnecessary thought operations by virtue of ingenious substitutions. At the same time, syllogizing is exemplarily insured against errors to which content-centered thinking is constantly prone.

Thus it is a matter of construing logic so it can enter competition with modern mathematics in this

decisive point. In other words, it is a problem of transforming the rules of the syllogism in general into *rules of calculating*. And what does that imply? It implies that these rules must be so formulated that when applying them one need no longer think at all of the meaning content of the expressions to which they have reference. This insight into the real function of the rules of the calculus must be looked upon as one of the greatest lights that dawned on Leibniz and one of the most beautiful illuminations of the human spirit in general.

With this interpretation every objection which is to this day supported by the principle of the so-called nonquantification of qualities is nullified *a priori*. For, quantity does not enter any longer into Leibniz' concept of the rules of calculus. He himself designated the calculus to be created—the *calculusratiocinator* (logical calculus), as he called it repeatedly [176]—a calculus of qualities.[177] Such a calculus can only be obtained when we succeed in substituting for the natural language of thought an artificial language, in other words, when we succeed in inventing a symbolism with the aid of which the statements in question can be so presented or, to speak with Leibniz, so "represented," that when operating with them we no longer need to think of their meaning content.[178]

It is patent now that we have arrived at the famous *characteristica universalis*. What, then, is this *characteristica universalis*? It is a system of symbols of which we postulate the following:

1) Between the symbols of the system (in so far as they are not symbols for zero places) and what is thought (in the widest possible sense of the word)

there must exist an unambiguous relation which is reversible. That means that for every thing thought there must exist one and *only one* symbol—the "image" of the thing thought—and, *vice versa,* for every symbol there must exist one and *only one* thing thought: Let us call it the "meaning" of the symbol.[179]

2) The symbols must be so devised that wherever a thing thought is present, which may be divided into its components, the "image" of these components must again be components of the picture of what is thought in the available symbols.

3) A system of operational rules auxiliary to these symbols must be invented so that wherever a thing thought T_1 stands to a thing thought T_2 in the relationship of antecedent-consequence, the "image" of T_2 may be interpreted as a consequence of the "image" of T_1.[180]

It is in view of the tremendous material which is extant today for Leibniz' logic [181] but which we cannot discuss here, and by virtue of the fact that we have many years' study to our credit, that we may be permitted to split up the following magnificent passage by Leibniz into the above given three components: *Ars characteristica est ars ita formandi atque ordinandi characteres, ut referant cogitationes seu ut eam inter se habeant relationem, quam cogitationes inter se habent. Expressio est aggregatum characterum rem quae exprimitur repraesentantium. Lex expressionum haec est: ut ex quarum rerum ideis componitur rei exprimendae idea, ex illarum rerum characteribus componatur rei expressio.*[182]

If we add that, according to the way things were set up, the operational rules postulated for operating

with the feasible combinations of symbols must function as rules for operating with these and *only* these symbols, then we can readily see in what sense and with what justification Leibniz himself, as the first, was able to interpret these rules as "rules of the game" and the result of the logic he had in mind as a reduction of logical operations to an "interlude" (*jeu de caractères*).[183]

"So much the worse for logic!" someone will exclaim, "in the beginning there was dead seriousness!" Such continually recurrent voices, however, cannot drown us out because the philosophic ethos which seems natural to the advocates of gravity exists only for those who, first of all, are not acquainted with the tremendous labor that has to go into the construction of a "playful" logic and, secondly, are either not *able* or not *willing* to realize that the quality of any type of logic is to be known by its fruits, and *only* by its fruits. Here also Leibniz is the great master from whom we can always learn so much. For it was he who led by holding up the criterion, the only one at that, which up to now has stood up under careful checking. He required of the new logic that it provide an infallible guide through the labyrinth of conclusions hidden in any and all systems of somehow meaningful premises. *"Filum cogitandi voco Methodum quandam facilem et certam, quam sequendo, sine agitatione mentis, sine litibus, sine formidine errandi, non minus secure procedamus, ac is, qui in labyrintho filum habet Ariadneum."* [184]

He had a vision of an approaching golden age in which, on the basis of the new logic, philosophical problems would be as "thoroughly calculated" as had

become possible for mathematical problems since the discovery of modern analytical methods. In that golden age it would be possible to assert for every genuine problem of metaphysics what Hilbert maintained for mathematics in the year 1900 at the Paris meeting of mathematicians, that every genuine mathematical problem must be soluble, and that so far as mathematics is concerned, there is no *ignoramus*. *"Id . . . efficiendum est, ut omnis paralogismus nihil aliud sit quam error calculi . . . Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo . . . dicere: calculemus!"* [185]

It will not be necessary to comment on how far we are still from this magnificent goal. Today we are perhaps more distant than ever before. Nevertheless it is necessary to ask whether we would not be a trifle further along if we had listened somewhat more attentively to Leibniz and somewhat less to Kant whose almost canonical criticism of the idea of a thoroughly mathematized metaphysics [186] in Leibniz' sense is one of the gravest reverses which logic had to suffer at Kant's hand. For, granting him everything else, there still remains always the possibility of a metaphysics in the if-so style [187] which was likewise already envisaged by Leibniz, hence the possibility of a metaphysics for which the expedients of Leibniz' logic can be nicely invoked.[188]

It is true, Leibniz did not leave us a complete logical system. What we do have are essentially only the magnificent fragments on the basis of which we can reconstruct his conception of this type of logic.

The execution has become a very big enterprise by itself, an undertaking of at least the same order of magnitude as the carrying out of Schelling's systematic program by Hegel. This task was undertaken and does exist since 1910 in the three volumes of the *Principia Mathematica* (Cambridge, vol. I, 1910; vol. II, 1912; vol. III, 1913) [189] which Bertrand Russell (1872-) brought into being with the assistance of Alfred North Whitehead (1861-1947).

Between Leibniz and Russell there lies a tremendous amount of labor of which only the most important phases can be touched upon. In the 18th century and still under the influence of Leibnizian ideas, Lambert and Gottfried Ploucquet (1716-1790), Hegel's teacher at Tübingen, worked on the construction of the logical calculus.[190] Then, for a time, leadership passed to the English. Quite independently of Leibniz and the German research work of the 18th century the two English mathematicians Augustus de Morgan (1806-1878) [191] and George Boole (1815-1864) [192] invented around the middle of the 19th century a new logical calculus which later was expanded by the German mathematician Ernst Schröder (1841-1902) into a grandiosely planned *Algebra der Logik*.[193] Since 1889 we meet the new type of logic with basic improvements in the works of the Italian mathematician G. Peano who did a great deal for the axiomatization of arithmetic. For the first time the most important propositions were presented by him in symbolic notations for larger and larger areas of mathematics.[194]

Unquestionably the greatest genius of modern logic of the 19th century was, however, the German

mathematician Gottlob Frege (1848-1925).[195] More than anyone else he contributed to the interpretation of basic mathematical concepts in terms of the fundamental concepts of logic which operate with exact determinations right from the start. The first one to do so, he raised the logical calculus to a level at which it turns into the "interlude" of which Leibniz had spoken. Nevertheless, he did not exert a direct and definitive influence, but in a roundabout way he did so by way of Russell's masterwork. The reason for this was that in spite of his thorough reflections he was not able to find the type of plastic symbolism which we have to postulate for a "conceptual script." In this great task only the authors of the *Principia Mathematica* succeeded. With the appearance of this opus the new logic was called into being. Since the joint proposal of Couturat, Lalande and Itelsohn at the 1904 International Congress of Philosophy in Geneva, it was more and more referred to as *"logistique"* or "Logistik" (symbolic logic). It was not until then that one had arrived at last at a preliminary system of *figures significantes par elles mêmes,* as Leibniz, the greatest theorist of symbolism in general, had demanded.[196]

B

What is it that this new logic is accomplishing? It accomplishes so much that we can't even think of enumerating its merits completely. We shall, therefore, limit ourselves to those achievements which can be formulated in such a way that knowledge of symbolic logic is not necessary for understanding the sen-

tences we are using. Even among these easily understandable achievements we can only name the most important ones. *Apropos* we should state the following.

1) Symbolic logic is the first type of formal logic which is *stylistically pure*. To be more specific, it is the first type of logic which is so exclusively concerned with the "perfect forms," [197] and the syllogistic rules deducible from the generally valid one among these "forms," that it deals with the rest of logical material, mainly concepts and judgments, only to the extent required for an understanding and development of the syllogism. Everything else is eliminated, and the whole ballast of psychology is completely cast off in symbolic logic. Psychology had weighed down in so-called formal, nonsymbolic logic the theories of concepts and judgments and had worked to great disadvantage for the proper task of this type of logic. Those questionable additions which classical formal logic was gradually "enriched" by since the days of the *Logic of Port Royal,* now decidedly disappear. Symbolic logic no longer needs such alleged enhancing; for it is so busy regulating the syllogism and has so much that is of interest to say that it does not stand in need of looking around for other things that are more attractive. Nor need it fear a *testimonium paupertatis* which some might be inclined to hurl at it. That alone is a merit very much worth mentioning. We owe Goethe the saying: Form, for most, is a secret. Now, for logic this form has been revealed in such purity by symbolic logic that one may speak of a paradigm in Plato's sense.

2) Symbolic logic is the first *exact* formal logic. It is exact in so far as in it the following beautiful characteristics stand out:

a) It is the first logic which defines its axiomatic material so precisely that it can be surveyed with the same clarity as the axioms of mathematics.

b) It is the first logic which formulates precisely the rules by whose correct application those propositions and concepts are to be obtained which differ from axiomatic propositions and basic concepts.

By virtue of a) and b) this logic is subjected to control which makes all inadmissible assertions and, likewise, all inadmissible concepts humanly impossible and provides a degree of certainty which may well be designated as admirable.

c) By virtue of the symbolism with which symbolic logic works, the statements of logic have become for the first time as precise as are the statements of mathematics. The full significance of this can only be realized by one whose mind has been whetted by symbolic logic to perceive the inexactitude of nonsymbolic formal logic. It may suffice to recall the so-called principles of logic in the shape in which they have been transmitted since Leibniz, say the principles of identity, of contradiction, of the excluded middle, and of sufficient reason. Though Leibniz is not to be blamed it may be said upon thoroughly going over the material that for the first time the first three of these principles were formulated by symbolic logic

in such a way that they can now be utilized with exactness for inference, and that many formulations that had been given them, especially the one given the principle of identity, are not only inexact to the point of unusableness, but are downright meaningless. With respect to the fourth principle, that of sufficient reason, we need say no more in this place than that it cannot really be placed on the same level with the first three principles. The positive reason is that it belongs to those things which cannot be formalized at all, thus cannot be expressed in symbolic notation at all, but can only be *formulated.* For, this too is one of the great merits attributable to symbolic logic that for the first time we can distinguish clearly between what can be formalized and what can only be formulated with respect to logical data. The principle of sufficient reason can, indeed, only be *formulated,* let us say in this way: In a strict science every proposition which differs from the premises must be grounded sufficiently, which can only mean these premises. It is immediately clear, then, that this principle does not belong to logic at all but is part of theoretical science.

A particularly beautiful proof of the precision attainable with the new symbolism is the symbolization of Brouwer's logic by A. Heyting.[198] It is by virtue of this symbolism that Brouwer's logic has become accessible at all to an exact judgment by those who hold different opinions.

d) In the report on the Ramist extension of Aristotelian logic [199] we were already stressing the formal inadequacy of this extension. We did emphasize that the new syllogistic rules were not formulated as such

at all, but were only explained by examples. The same must be said of the extensions of Aristotelian logic by the *Logica Hamburgensis*.[200] Symbolic logic is the first type of formal logic which is capable of really formulating these non-Aristotelian syllogistic rules whose number is legion. Specifically it is, thus, the first type of logic which provides the precise syllogistic rule required of a syllogism: *Circulus est figura; ergo quicunque circulum describit, is figuram describit.*[201] Let anyone try to formulate once this rule in this manner and he will no longer have to strain himself to work up a respect for a logic which can work out such and even much more complicated rules by the hundreds.

e) It was further brought out earlier that Aristotelian logic is not able to describe *syllogizing*.[202] What we are really doing when we syllogize we have come to know with precision only through symbolic logic.[203]

f) Symbolic logic is the first formal logic which has furnished a precise analysis of the *copula*. It did so by orienting the manifold meanings of the copula strictly by the only point of view which is essential in formal logic: the influence it has on the formulation of syllogistic rules. It is astonishing to the point of incomprehensibility how little classical formal logic has done for the logical analysis of the copula, in spite of the famous Aristotelian proposition regarding the manifold meanings of being; with respect to logic, more specifically, the little word "is." Which logic, prior to symbolic logic, has shown that the "is" in "7 is a prime number" is totally different logically from the "is" in "the raven is black" and just as

completely logically different from the "is" in "Goethe is the poet-author of *Faust*"? Not any of them! Only symbolic logic has furnished the demonstration, and it did so by pointing out that for these three kinds of "is" totally different syllogistic rules apply.

In this connection we must also mention that symbolic logic is the first formal logic to furnish the symbolic support for a clear-cut interpretation of statements of the form: "All S are P," "Some S are P," and "S is P," and, hence, has made all theories of judgment antiquated which try to achieve a reduction of the statements of one of these three forms to the predication of a relation, identical for all three, between the intension (or extension) of a subject concept and the intension (or extension) of a predicate concept. It has shown that all theories of judgment and, as a result, perhaps all theories of judgment in qualitative formal logic in general, are in the strict sense false, that they furnish wrong conclusions even when consistently applied.[204]

g) Symbolic logic is the first formal logic which has produced an exact, logical analysis of existence, with Kant's condition in mind that existence can never be predicated of individuals.[205] For the first time it provides a clear-cut answer to the problems that arise when we may infer *as to* existence and which concern what we may infer *from* existence. It likewise tells us with an exactitude we may consider exemplary what we have to understand by the existence of a property and what by existence of a relation. Thus it puts an end to the problems in logic which Lotze was able only to formulate when he spoke of a "value" of properties in contrast to the existence of things *with* these properties, and, similarly, of the

"being" of relations, in contrast to the existence of things *with* these relations. What contributions the more recent value logic (in Rickert and, above all, his highly gifted pupil Lask, who bested himself in inaccessible abstractions) which follows Lotze in these matters has made to an analysis of these forms of existence may, let us concede on other grounds, be important for other reasons and worth giving serious consideration to; but for formal logic we need *not* take notice of it. Only the pronouncements of symbolic logic are of consequence for formal logic in these matters. And, once and for all, let it be said right here and now that it is not certain symbolic logic blinders and certainly not symbolic logic arrogance which have dictated the sentiments of this little volume. Rather it is the same feeling of responsibility which first made Aristotle sit in judgment over the effectiveness of the logic of his great master Plato. It was not easy for him to launch his criticism. The reader may believe that it was not any easier for us either!

h) One more thing must be stressed here. In consequence of the interpretation by symbolic logic of the higher forms of existence as we wish to call them briefly, the famous conflict between nominalism and realism has been decided in favor of nominalism—thus against Plato and in favor of Aristotle. The precise significance of this is that a property may be predicated; likewise, a relation "exists" when, and only when, a system of things exists of which this relation may be predicated.

3) Symbolic logic is the first formal logic which has been developed *strictly synthetically*.[206] In other

words, it is the first type of logic which methodically ascends from the simple to the complicated in the sense already envisaged by the Stoics.[207] When we first studied the Stoics we stressed the point that a logic thus construed must start with a logic of statements and it is only then that the Aristotelian logic of predication can be built on such a basis. Symbolic logic has done this in an exemplary fashion; specifically during its development it has outfitted the Aristotelian logic of predication with all the enrichments of which it is not only capable but, above all, stands in need.

Beyond that, symbolic logic has succeeded, and succeeded for the first time in the history of logic, to construe logic in such a way that the age-old conflict between the logic of intension and the logic of extension has been eliminated for all time, so far as we can humanly judge. It has been able to develop logic in such a way that both the logic of statements and the logic of predication together furnish basically everything we require in an exact logic. If we designate it as a logic of intension, to use the common expression, then the priority has been decided by symbolic logic in favor of the logic of intension. More than that, it has even arrived at the decision that, in principle, the logic of extension is dispensable.[208] Viewed from another angle, symbolic logic has likewise recognized the practical *usefulness* of the logic of extension so clearly and defined it so sharply as no other logic has done previously. It has recognized the fact that the incomparable practical value of the logic of extension is to be looked for in its stenographic function and in this sense has developed a magnificent theoretical structure of this logic outdistancing everything that has gone before.

4) Symbolic logic is the first *perfect* formal logic. Expressed in other terms, it is the first logic, about which we can assert that it gives us the complete inferential rules which the development of the tremendously exacting modern mathematics requires. The proof of this assertion has been given by Russell in his opus by actually developing the fundamentals of modern mathematics with symbolic logic material. Hence the title of *Principia Mathematica.*

Now we should not say that this method is, thus, only adequate for mathematics. For, somewhere a beginning must be made, and Aristotle brought his theory of science into being by orienting it in exactly the same manner.

A general remark is now in order. Of all evils with which present-day philosophy is afflicted, one of the greatest is lack of self-restraint. Too much is being demanded and hence one is forced to philosophize really so sloppily while seemingly fulfilling these demands, that one cannot be held accountable because and only because of this sloppiness. How irresponsible is the work done today with quotation marks in all fields, in logic more than anywhere else. Every quotation mark is an indication of something left undone. Though of late we have forgotten this entirely, we shall rediscover it; even Plato and Aristotle knew that no thorough work is accomplished in this way. Worth-while work can be done even now in the field of philosophy if we have the courage of limitation and the courage to analyze. It can be said of contemporary logic that a platform has been provided on which future generations can build, just as was done in mathematics. *"Pauca, sed matura"* was the great motto of Gauss. It is also the motto of the new logic,

and if anything remains to be desired it is that a new brand of philosophizing is kindled by this logic. Only then will Leibniz' dream be realized, only then will be accomplished in philosophy strictly and centrally speaking what is worth the pains and the sweat of noble minds.

We should emphasize at this point earnestly and emphatically so that nobody will fail to hear it: We are still far distant from a realization of the magnificent idea which is responsible for Leibniz' conception of symbolic logic. There is thus far only a single example of the application of symbolic logic to a nonmathematical field which has stood up under careful checking, and that is the work of Rudolf Carnap, *Der logische Aufbau der Welt* (1928). This work, it must be admitted, is at all events a highly respectable accomplishment. It stands way above the average philosophical literature because it has been thought through most carefully. Nevertheless, in the preface at least, it wages a battle against metaphysics on the authority of Mach and in the name of scientific philosophy from which I personally would like to dissociate myself. Of course, I am not mentioning this in order to discourage the reader from studying this work. On the contrary, for the study of it can go a long way toward making one realize what is demanded nowadays of a serious and thoroughly developed philosophical standpoint. However, it has not yet been decided at all whether a scientific *Real-Philosophie* can be established *only* in a combination of symbolic logic with a positivism such as Carnap professes. It may even more strongly and justly be doubted that a problem which does not exist for this combination reveals by that very fact alone its illusory nature. I

would rather be inclined to say that it is very much to be regretted that the "Vienna Circle," so impressively represented by Carnap, has vitiated itself by making such a problematic statement. Very emphatically I would like to add that I have no doubt that Leibniz has even in this matter seen much farther than these positivistic Leibnizians whom we must, of course, reckon with in their own field. It has certainly not been established for certain that a convinced symbolic logician may not at the same time be a metaphysician in the strictly defined Leibnizian sense. He is a thinking human being for whom even the problem of God exists as a grave philosophical problem; and that may not be set aside so easily, not even by a positivist however distinguished. One should take care, therefore, not to paralyze the central philosophical potential of symbolic logic which we, together with Leibniz, claim, by insisting that the new logic be intimately coupled with an extreme positivism such as manifests itself currently in some of the strongest proponents of logical positivism.

One more thing must be added. For the members of the Vienna Circle it is already a proven fact that a so-called assertion is only then a meaningful statement if it may be expressed in the symbols of the axiomatic material of Russell's logic. Let me make the following pertinent remarks.

a) As a maxim this positive requirement is well and good, for it obligates us, by putting it this way, to extract from symbolic logic all that can be extracted. As to the limits we can come to a decision only after having increased the capacity of this logic to the breaking point. Even logicians who do not sub-

scribe to symbolic logic we can, since Kant's critique of reason, require to acknowledge this.

b) As soon as one transcends these limits this positive requirement becomes dictatorial, and against dictatorship in philosophy even the most convinced symbolic logician should protest to his dying breath.

c) Thus, we are *not* making the assertion that logic in Leibniz' sense has reached its perfection in present-day symbolic logic. Not in the least. We are only averring, but with a determination unafraid of full investigation, that symbolic logic has laid the groundwork on which we can *continue to build* so that every other philosophical discipline may see clearly how a foundation is to be laid.

d) Just as scarcely do we assert that mastery of symbolic logic is in itself sufficient to make one a philosopher. While defending symbolic logic we encounter again and again this abysmal misconception among those who are not disciples of symbolic logic. We are asserting merely that mastery of symbolic logic is certainly at all events desirable today in a philosopher who has interest in philosophizing scientifically, and that even a moderate training in the rigor and acumen of logical thinking has become necessary for one's being a philosopher of science. We did say necessary, not merely requisite.

The study of symbolic logic is absolutely necessary. However, we do not intend to say that the new metaphysics must from now on be expressed in notations of symbolic logic. We cannot be downed that easily. What we mean is that a contemporary metaphysics requires the severity and meticulousness of thought by

which we recognize a disciple of symbolic logic even *outside* logic itself. We demand no less, no more. That a great deal of what continues highfalutin in metaphysics will disappear with this austerity must be reckoned a gain and not a loss. Nobody can prognosticate how much of what is novel will appear on the scene. Just remember the critique of reason. Not one pre-Kantian interpreter of reason, up to Leibniz, could state even remotely about its *potentialities* what the greatest critic of reason was able to say about its merits.

5) Symbolic logic is the first *experimental* logic or, in other words, the first type of logic which has also investigated systems of syllogistic rules which are *not* identical with those of Aristotle. The light that Brouwer's logic has cast may once again be mentioned as an example. Other investigations with much higher goals are currently being made. These highly interesting achievements have nullified automatically the arguments marshalled by qualitative logicians of all shades in support of an alleged impossibility of such investigations.[209]

6) And now let us appreciate what symbolic logic has liberated us *from*. This problem, too, we have at least to touch on. Think of the fight against psychologism in logic which has already lasted a generation under Husserl's leadership. It is a battle that has to be waged, to be certain; but how few are the gains for positive logic outside symbolic logic! Not even the "principles" of logic, so far as I can see, have yet been formulated satisfactorily by the opponents of

psychologism who are not at the same time adherents of symbolic logic. Symbolic logic has tackled the job quite differently. It has symbolized logic to such an extent that a psychological interpretation of the symbolized expressions is *a priori* impossible, just as impossible as the normative interpretation which still raises its head. It is, of course, very obvious that these and other curiosities continued to exist for such a long time and still engage attention because logic itself had not been raised to the level to which symbolic logic has now lifted it.

The following may likewise be considered a feat of liberation of the first magnitude: Symbolic logic has shown that for further reconstruction in mathematics at any rate one does *not* require Aristotle's exceedingly penetrating but highly complicated *logic of modalities.* It is possible, however, that in a not too distant future a new logic may develop on Aristotle's theory of probability. It has been shown further that one can construct a logic before having threshed out the *ontological problems* with which classical logic has been weighed down since its inception by Aristotle, so weighed down, in fact, that it has not been able to rise for that very same reason. Symbolic logic has shown us that we can develop logic so that it makes absolutely no difference whether one interprets the principle of contradiction, to name only the most famous example, saying: "Every statement which attributes to an individual with a predicate P at the same time the predicate non-P which is contradictory to P, is false," or "There is no individual with a property E to whom simultaneously the property non-E contradictory to E, belongs." What rivers of

ink have not flowed in the field of classical logic merely for the purpose of discussing this disputed question! Nothing was accomplished in all this, at least not until recently.

Finally and above all symbolic logic has shown that one can liberate logic from the unbearable pressure of the *problem of evidence,* liberate it in the sense that one chooses the axioms in such a way that they are fairly intelligible or plausible. Then one extracts from them everything that with the aid of a thoroughly worked out system of meaningful rules can be extracted from them. This may be done instead of wearing oneself out trying to solve the insoluble problem as to what the feeling of evidence rests upon and with what criteria it is to be furnished. Let it be understood, however, that we do *not* belong to those for whom the problem of evidence does not exist at all, and more emphatically that we do not by any manner or means belong to those who repudiate a problem in logic *because* it has been formulated by one who is not a follower of symbolic logic. For that would demonstrate an arrogance which we have already openly rejected. Yet it must nevertheless be said by anyone who is eager for results that in the discussions of this problem mighty little has been gained to this day. Not even *freedom from* contradiction was gained for classical logic. On the contrary, even this deepest concern was turned over to the disciples of symbolic logic who then were derided for the tremendous pains they expended on it.

7) It was not merely the absence of contradiction in logic which symbolic logic tackled for the first time seriously, but symbolic logic took on other problems

which are very important for correct inference yet were neglected completely in the classical logic. We are thinking of the analysis of the *definite article* in statements of the form "The so-and-so." [210] An exact logical analysis of these expressions is an absolutely essential and preliminary task for the formulation of correct rules of inference. Up to now classical logic has not even had an inkling of the fact that here we are faced by a considerable task which lies within the logician's competence.

However, above all we are thinking here of the *exemplary technique of negation* which symbolic logic has developed. It is a practical present which anyone will gratefully accept who now and then finds himself in the position of being obliged to properly negate some given propositions. Even the simple case of the Euclidean parallel postulate may serve as an illustration. In its most convenient formulation it states: "To any straight line and through a point *not* lying on it, there may be drawn at the most ($=$ not more than) one parallel line." How does the contradictory negation of this Euclidean parallel postulate read? It states: "There is at least one straight line with respect to which there exists at least one point *not* lying on it suchwise that through this point there passes at the most *not* one, thus *not* not more than one, and hence more than one parallel (to the given straight line)." On the basis of long experience I believe I can state that the number of cases is rather small in which a qualitative logician will arrive at the formulation without going astray at least a couple of times, sometimes rather seriously. Symbolic logic puts this negation right into our lap, and it does so

for every meaningful case. Complicatedness is of no concern. On the contrary, the more complicated the expression to be negated the more ingeniously does its technique operate.

By way of conclusion we may say that in the work done by symbolic logic thus far so much has turned up in every case of application that we may speak of a generous gift, a harvest in the fullest sense of the word. That is the reason for the emphasis we have put on it. Only since the appearance of symbolic logic can we speak with justification of a history of logic as we have done in this little volume. To this very day we cannot with equal justification speak of a history in any other field of philosophy.

BIBLIOGRAPHIC APPENDIX

A. The Types of Logic

Karl Rosenkranz: *Die Modifikationen der Logik, abgeleitet aus dem Begriff des Denkens* (1846),[211] is a work which occupies the standpoint of Hegel: "We intend to explain every position from the concept of thought for any special form of logic by an example from history. This we do for the purpose of increasing the certainty that we are not merely dealing in abstract deductions but are demonstrating that real thought differences do exist in the notion. Such differences have had the power of becoming factors in the concrete development of science" (p. 175 f.).

If I understand Edmund Husserl correctly it is he who in his work *Formale und transzendentale Logik, Versuch einer Kritik der logischen Vernunft* (1929) projected a modern abstract morphology of logic, or, rather, the prolegomena to such a one.

Joh. Baptist Rieffert has attempted a concrete morphology of logic in the work entitled *Lògik, eine Kritik an der Geschichte ihrer Idee* (1925),[212] putting

the emphasis on the 19th century. He does so by assuming Erdmann's standpoint, thus placing himself in a position diametrically opposed to the one we represent in this concise history of logic. But for this reason it is doubly recommended to the critical reader who can thus expand his mental horizon. In this book of barely 300 pages the following number of pages have been allotted to the different thinkers: 35 to Erdmann; 13 to Sigwart; 7 to Aristotle; ½ to Leibniz, the algebraizing logician (!); 2 to Bolzano. The volume is thus at all events not only an antithetical counterpart but also a complement to the history presented here.

Very interesting and at any rate worth mentioning here in the larger context is the great work by Hans Leisegang: *Denkformen* (1928). To be sure, it does not in the main deal with the theorists in logic but with the practitioners. Those practitioners are preferred who, for Aristotelians, are either highly problematic (Paulus) or are not directly accessible at all to them (Hegel).[213] However, by its very title it comes so close to the problems of morphology that we have to call attention to it without fail. It should be noted that in the arguments that are marshaled the principle of the possibility of existence of *different* formal logical systems (whose legitimacy may be deduced from reasons quite different from those the author supplies) or, as Leisegang expresses it, we think rather unhappily, the principle of the fictional character of the One Logic,[214] has *not* been proven. On the contrary, in his arguments he is operating with quite a muddled concept of formal logic. Were he to place the emphasis in his thinking on precision, he would have to substitute his arguments by others than the

ones he is giving. Yet, let us consider this rather unessential here in view of the abundant and thorough treatment he accords the material which he does lay before us.

B. The History of Logic

Apart from the standard work of Carl Prantl already mentioned and described in the Preface, *Geschichte der Logik im Abendlande* (4 vols., 1855-1870, available since 1927 in an excellent reprint from Gustav Fock, Leipzig) we must mention above all the Cyclopean collection of notices and articles by G. Vailati (1863-1909): *Scritti* (Firenze-Leipzig, 1911),[215] which have proven to be of value in clarifying many important details in the whole history of logic. Very useful and easy to read is the many-sided and suggestive little book by Federigo Enriques: *Per la storia della Logica. I Principii della scienza nel concetto dei pensatori matematici* (Bologna, Zanichelli, n.d.) written in a vivid style and based on his own *aperçu*. It is well that it has been translated into German by Ludwig Bieberbach as *Zur Geschichte der Logik, Grundlagen und Aufbau der Wissenschaft im Urteil der mathematischen Denker* (1927),[216] and readably at that. Finally, an outline of the history of logic much too little known in Germany is the small volume by Robert Adamson: *A Short History of Logic,* edited by W. R. Sorley (Edinburgh and London, 1911). For the field of pre-Aristotelian logic which we have not included in this history, it is best to consult Julius Stenzel's article "Logik" in Pauly-Wissowa's *Real-Encyclopädie der klassischen Altertumswissenschaft,* 25th half volume (1926), columns 991-1011.

Very full, very dependable and very interesting is the historical material treated in Bolzano's four-volume *Wissenschaftslehre* (1837),[217] particularly in the first and second volumes. In connection with that we should mention the exemplary and meticulously prepared register of authors by Wolfgang Schultz at the end of volume four. Also in Benno Erdmann's *Logik* which appeared in a third edition in 1923, historical material of quite considerable volume has been dependably assembled or at least indicated. Likewise, but not always with the same reliableness[218] we find extensive historical material in the voluminous work of Theodor Ziehen: *Lehrbuch der Logik auf positivistischer Grundlage mit Berücksichtigung der Geschichte der Logik* (1920). The general history of logic treated in this work in 200 pages which are, for the most part, closely printed, is a very useful bibliography which we recommend warmly for a preliminary orientation. It supersedes essentially the corresponding treatment in the well-known work by Friedrich Ueberweg: *System der Logik und Geschichte der logischen Lehren* whose 5th edition was revised and edited in 1882 by Jürgen Bona Meyer and is still quite useful as an introduction.

The works of Venn, Lewis, and others may be consulted in Appendix E.

C. Aristotle

Though no longer adequate to the demands of logicians so far as interpunction and commentaries are concerned, *Aristotelis Organon Graece, ed.* Theodorus Waitz (I, 1844; II, 1846) is still the best edition. Having been out of print for a number of years, the

study of Aristotelian logic has been made difficult in the extreme merely for technical reasons. Only the *Topica* together with the *Sophistici Elenchi* have, in the meantime, been reissued by Strache-Wallies.[219] Under these dire circumstances the excellently printed and cheap English edition of selections: *Selecta ex Organo Aristoteleo Capitula, in usum scholarum academicarum* (Oxford, Clarendon Press, 1902) still does valuable yeoman service. This little book is at any rate to be preferred in my judgment to the selections with comments by Adolf Trendelenburg[220] which, as everyone knows, was most valuable in his day even though he did not collect the essentials exhaustively. It, too, has been out of print for some time.

All the more valuable is the very dependable English translation of the *Organon* which was prepared under the direction of the highly deserving English Aristotle scholar W. D. Ross: *The Works of Aristotle, translated into English.* I—*Categoriae* and *De interpretatione,* by E. M. Edghill; *Analytica priora,* by A. J. Jenkinson; *Analytica posteriora,* by G. R. G. Mure; *Topica* and *De sophisticis elenchis,* by W. A. Pickard-Cambridge (Oxford, Clarendon Press, 1928).

Of ancient interpretations still indispensable today may be mentioned the commentaries by Alexander of Aphrodisias[221] that have come down to us: *In Aristoteles Analyticorum priorum librum I,* edited by M. Wallies (1883),[222] and *In Aristotelis Topicorum libros octo,* edited by the same (1891).[223] Of later interpretations most valuable still is the standard work of Julius Pacius a Beriga (1550-1635): *Aristotelis . . . Organum* (1584).[224] The latest comprehensive inter-

pretation has been given in the well-known work of Heinrich Maier's: *Die Syllogistik des Aristoteles,* I (1896): *Die logische Theorie des Urteils bei Aristoteles;* II (1900): *Die logische Theorie des Syllogismus und die Entstehung der Aristotelischen Logik.* Let me add the article on "Syllogistik" by E. Kapp in Pauly-Wissowa's *Real-Encyklopädie der klassischen Altertumswissenschaft,* 2nd Series, 7th half volume (1931), columns 1046-1067, as a valuable philological supplement. The most exact interpretation of the Aristotelian *modi* has been furnished us so far as I can judge, by Kazimierz Ajdukiewiecz: *Zalozenia logiki tradycyjnej,* in *Przeglad Filosoficzny,* vol. 29 (Warszawa, 1926), pp. 200-229; *Kant-Studien,* vol. 34 (1929), p. 410 f.

D. Introduction to Present-day Logic

J. Klemens Kreibig: *Die intellektuellen Funktionen. Untersuchungen über die Grenzfragen der Logik, Psychologie und Erkenntnistheorie* (1909). The surveys which the reader may unexpectedly discover in this work are still *very* useful today. And Kreibig knows who Bolzano was!

Windelband-Ruge: *Enzyklopädie der Philosophischen Wissenschaften.* I: *Logik* (1912), including six relevant treatises by Wilhelm Windelband, Josiah Royce,[225] Louis Couturat,[225] Benedetto Croce,[226] Federigo Enriques, and Nikolaj Losskij.

Wilhelm Koppelmann: *Untersuchungen zur Logik der Gegenwart* I (1913): *Zur Lehre vom Denken und Erkennen;* II (1918): *Formale Logik,* is a work written from the Kantian point of view.

Joseph Geyser: *Grundlegung der Logik und Er-*

kenntnistheorie in positiver und kritischer Darstellung (1919), and *Auf dem Kampffelde der Logik* (1926), two very good orientations regarding the modern position of nonsymbolic logic.

E. Symbolic Logic

Apart from the handbooks by Whitehead and Russell as well as Hilbert and Ackermann which are mentioned in the text [227] we have to refer principally to the excellent *Abriss der Logistik, mit besonderer Berücksichtigung der Relationstheorie und ihrer Anwendungen* by Rudolf Carnap (1929); [228] furthermore, the little volume by Heinrich Behmann: *Mathematik und Logik* (1927) [229] which is very useful to an advanced student. The one proficient in the logic of statements and predication will find J. Herbrand: *Recherches sur la théorie de la démonstration* (1930) [230] very instructive. Poland has lately become the main country and Warsaw the main bastion of research in symbolic logic by virtue of the work of Jan Lukasiewicz.[231] We can only refer to the pertinent treatises by Stanislaw Lesniewski, W. Sierpinski, Alfred Tarski and others in the *Fundamenta Mathematicae* of which volume 16 appeared in Warszawa during 1930. They all are geared to undergirding the foundations of mathematics. Also Leon Chwistek: *The Theory of Constructive Types, Principles of Logic and Mathematics* (Cracow, University Press, 1925) must at least be alluded to.

Bertrand Russell's *Introduction to Mathematical Philosophy* which appeared in London in 1919 and has since passed through many editions must still be given first place among *introductions* to symbolic

logic. For purposes of thorough study the German translation by E. J. Gumbel and W. Gordon, under the title of *Einführung in die mathematische Philosophie* (meaning, of course, the philosophy of mathematics), which appeared in 1923 [232] is, unfortunately, not reliable enough to be taken as a substitute for the perusal of the original. Aside from this book we note R. Feys: *La transcription logistique du raisonnement, son intérêt et ses limites* [233] and *Le raisonnement en termes de faits dans la logistique Russellienne,* [234] as worth reading. As a matter of course the new logic, too, is freighted with metaphysical and epistemological presuppositions, yet fortunately to a much lesser degree than any of the earlier formal logical systems. Raymond P. Hawes: *The Logic of Contemporary English Realism* (New York, 1923) [235] gives us valuable information metaphysically and epistemologically in the case of Russell's logic. At all events, a very high level—probably more or less inaccessible for most students even today—has been struck by the *Tractatus Logico-Philosophicus* of Ludwig Wittgenstein (German, with English translation, London, 1922) which, though critical in many instances of Russell's logic, has been adjudged by Russell himself as the work of a genius.[236]

It behooves us to call attention among more recent publications to the special number edited by Adolf Fraenkel of the *Blätter für Deutsche Philosophie,* vol. 4 (1930-31), Number 3/4: *Philosophische Grundlegung der Mathematik,* published by Karl Alber (Freiburg and München), and the collected papers of the keen and sensitive English thinker of whom science has been deprived much too soon, Frank Plumpton Ramsey: *The Foundations of Mathematics and Other*

Logical Essays, edited by R. B. Braithwaite, with a preface by G. E. Moore (London, 1931).

Thus far the most striking *application* of the new logic is Hilbert's *Grundlagenforschung.* The best introduction we have is the Hilbert lectures which were reprinted in the 7th edition of the *Grundlagen der Geometrie* (1930)[237] as Appendices VII to X. Anent Hilbert's endeavors there appeared a highly stimulating article by Kurt Gödel in the *Monatshefte für Mathematik und Physik,* vol. 38 (1931), Number 1, under the title of *Ueber formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I.* If the results of this article withstand thorough investigation Gödel will have demonstrated that the expedients invoked until now by Hilbert are no longer sufficient for proving formally, as Hilbert tried to do, that our mathematics is free of contradictions. This holds true, moreover, of a whole slew of more comprehensive expedients which we can determine with exactitude. Th. Skolem, likewise, furnished a critical study of unquestioned competence in *Ueber einige Grundlagenfragen der Mathematik* (Oslo, 1929).

Very interesting further applications of symbolic logic and related thinking are encountered in the members of the so-called Vienna Circle. The instructive little book on *Wissenschaftliche Weltauffassung, Der Wiener Kreis* (Wien, Arthur Wolf, 1929) provides the best information regarding the objectives and accomplishments of this circle which is spearheaded by Moritz Schlick. Compare with this the very valuable bibliography in *Erkenntnis I* (1929), pp. 315-339. Quite informative with respect to the possibilities of application of the new logic is likewise Walter Dubis-

lav: *Die Definition* which appeared in third edition in 1931.

By far the most comprehensive *historical and critical exposition* of the new logic has been furnished by the Danish scholar Jörgen Jörgensen with his highly interesting and instructive three-volume work: *A Treatise of Formal Logic, its Evolution and Main Branches, with its Relations to Mathematics and Philosophy* (Copenhagen/London, 1931), vol. I: *Historical Development;* vol. II: *Systematic Exposition;* vol. III: *Discussion and Criticism.*

For the *history* of symbolic logic in a wider sense we have to name in the first place the excellent work by John Venn: *Symbolic Logic,* whose second edition appeared in London in 1894, and the work which is on just as high a level in its own way, C. I. Lewis' *A Survey of Symbolic Logic* (Berkeley, 1918). The latter is provided with a comprehensive bibliography and could also be designated as the best presentation of the "Algebra of Logic," the predecessor of symbolic logic.

SUPPLEMENTARY OBSERVATION

On Aristotle's Limitation of the Principle of the Excluded Middle (*exclusi tertii principium*) [238]

Moritz Schlick, in his very instructive essay on *"Die Kausalität in der gegenwärtigen Physik," Naturwissenschaften* (1931), pp. 145-162, has offered a criticism of Aristotle's restriction of this principle which in my opinion unquestionably warrants its being mentioned and discussed in an appendix to this history of logic. His criticism follows upon a clarification of the expression "determinism" in its compatibility with our present state of knowledge. The clarification is most illuminating. For it is true that when we ask what the proposition "Event E is determined" means, we should not think with old and new metaphysicians of some mysterious bond which connects E with other events. The existence of such a bond is assuredly incapable of being demonstrated by any known observation. We shall have to limit ourselves to explaining the determinateness of E by the *predictability* or *an-*

ticipatory calculability of E, for this is, indeed, a controllable predicate.

This insight is, according to Schlick,[239] sufficient "to dissolve a famous paradox important for the problem of causality, to which Aristotle already had fallen prey and which is causing confusion right down to the present." It is the paradox of so-called "logical determinism." It says that the principles of contradiction and excluded middle [240] would not hold true for propositions about future actual situations if there were no determinism. Actually (and Aristotle already argued this) if indeterminism is true, if the future is thus not already *determined* here and now, it would seem that the proposition "Event E will take place day after tomorrow" would be neither true nor false today. For, if it were true, for instance, then the event would have to take place, it would be already fixed contrary to the presupposition of indeterminism.[241] Even nowadays this argument is occasionally held to be cogent, indeed, has been made the basis of a novel type of logic (*cf.* J. Lukasiewicz: *"Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalküls," Comptes Rendus des Séances de la Société des Sciences et des Lettres de Varsovie,* 1930, pp. 63 ff.).[242] Nevertheless there must, of course, be an error here, for the logical propositions which are, it must be remembered, only rules of our symbolism,[243] cannot, so far as their validity goes, depend on whether there is causality in the world or not: In every proposition there inheres truth or falsity as a timeless property.[244] A correct interpretation of determinism eliminates the difficulty at once and leaves the validity of logical principles intact. The statement

"Event E will occur on such and such a day" is timeless and, thus, true or false even at this moment. It can only be one of the two alternatives, quite independently of the fact whether determinism or indeterminism prevails in the world. By no means does indeterminism assert that the proposition concerning the future of E is not already, or today, unambiguously true or false, but only that the truth or falsity of that proposition cannot be *calculated* on the basis of the propositions concerning present events.[245] The result is that we cannot *know* whether the proposition is true until the point of time in question has passed, but with its being true or with logical principles all this has not the least concern.

NOTES

Preface

1. Concerning this by far the most important historian of logic after Prantl, *cf.* the beautiful *in memoriam* by André Lalande: *"L'oeuvre de Louis Couturat,"* in *Revue de Métaphysique et Morale,* 1915.

2. *KV,*[2] VIII.

3. I could also have said "the variety of meanings of the designation 'logic.' " I have avoided this phrase in the heading of this paragraph only because of its clumsiness.

4. See below, p. 30 f. *Cf.* the magnificent autocharacterization toward the end of the *Topica, De soph. el.* 33, p. 183b, 34 ff.:

ταύτης δὲ τῆς πραγματείας οὐ τὸ μὲν ἦν τὸ δ'

οὐκ ἦν προεξειργασμένον, ἀλλ' οὐδὲν παντελῶς ὑπῆρχεν.

p. 184a, 9f.:

περὶ μὲν τῶν ρητορικῶν ὑπῆρχε πολλὰ καὶ παλαιὰ τὰ λεγόμενα, περὶ δὲ τοῦ συλλογίζεσθαι

(for, an exact interpretation of the function of the syllogism and a detailed criticism of the fallacies, too, belong to the working and basic inventory of the *Topica!*)

παντελῶς οὐδὲν εἴχομεν πρότερον ἄλλο λέγειν, ἀλλ' ἢ τριβῇ ζητοῦντες πολὺν χρόνον ἐπονοῦμεν.

Just as surely as this characterization has reference, first of all, to the *Topica,* it may also be applied in Aristotle's sense to the *Analytica priora* with their splendid and thorough discussion of the syllogistic forms. All the more may justly be maintained for this accomplishment what Aristotle elsewhere says for the *Topica* p. 183b, 22f. and 25f.:

μέγιστον γὰρ ἴσως ἀρχὴ παντὸς

and

ταύτης δ' εὑρημένης ῥᾷον τὸ προστιθέναι καὶ συναύξειν τὸ λοιπόν ἐστιν.

5. More details in my treatise on the *Axiomatik der Alten,* in *Blätter für Deutsche Philosophie,* vol. 4 (1930-31), pp. 259-278.

6. *De interpret. c.* 4, p. 17a, 1 ff.:

ἔστι δὲ λόγος ἅπας μὲν σημαντικός, ... ἀποφαντικὸς δὲ οὐ πᾶς, ἀλλ' ἐν ᾧ τὸ ἀληθεύειν ἢ ψεύδεσθαι ὑπάρχει.

"Thus (by virtue of the preceding explanation) every statement has the property of possessing a meaning;

but not every statement has also the property of asserting something. (This property a statement has) only when truth or falsity are inherent in it." As an example of a nonassertive statement prayer (*'he euché*) is adduced in what follows.

7. Albertus Magnus: *De praedicabilibus, tract.* I: *de natura Logicae, c.* 1 (*Opera* I, *ed.* P. Jammy, Lyon, 1651, p. 1a): *Quidam antiquorum Logicam nullam esse scientiam contenderunt, dicentes non posse esse scientiam id quod est omnis scientiae sive doctrinae modus.*

Regarding the term *modus, cf.* Aristotle: *Met., A*3, p. 995a, 13f.:

> ἄτοπον ἅμα ζητεῖν ἐπιστήμην καὶ τρόπον ἐπιστήμης.

In this connection see Alexander: *In Aristotelis Metaphysica Commentaria, ed.* M. Hayduck, *CAG,* I (1891), p. 168, 24 f.:

> ἄτοπον γὰρ ἅμα ζητεῖν ἐπιστήμην τινός, καὶ περὶ αὐτῆς τῆς ἐπιστήμης ζητεῖν τίνα τρόπον γίγνεται.

"It is incongruous to search with a certain science simultaneously the mode of existence of science in general (or the kind and manner in which a science in general must be construed)."

With this remark Aristotle attacks, therefore, immediately only confusing science with the *theory* of science. However, after this only a few steps are required in order to arrive at the principle that logic is incapable of being represented as science in the strict sense. We shall find that Hegel in his *Science of Logic* (see below, p. 19) knows nothing at all about such deep-searching reflections. Instead he speaks all

the more distainfully of the kind of logic which takes great pains to look into such basic problems.

8. The Aristotelian opposite to

λογικῶς

seems to me to be the expression

ἐκ τῶν κειμένων

in its most general formulation.

Τὰ κείμενα

are in this case the specific presuppositions requisite in a scientific demonstration in the Aristotelian sense, apart from the correct use of syllogistic rules. *An. post.* I, 32, p. 88a, 18 f. and 30 f.:

> τὰς δ' αὐτὰς ἀρχὰς ἁπάντων εἶναι τῶν συλλογισμῶν ἀδύνατον, πρῶτον μὲν λογικῶς θεωροῦσιν... ἐκ δὲ τῶν κειμένων ὧδε.

This

ἐκ τῶν κειμένων,

then, becomes specialized in Aristotle as the need arises. In the *Analytica* he was able to substitute it by

ἀναλυτικῶς

so that we get the contrast:

λογικῶς—ἀναλυτικῶς

(*An. post.* I, 22, p. 84a, 7 f. and b 2). In the physical treatise *de gen. et corr.* I, 2

ἐκ τῶν κειμένων

he substitutes correspondingly by

φυσικῶς.

By contrasting the atomism of Democritus and Plato, more precisely the atomism of Democritus and Plato's pseudoatomism, we can gain a clear conception,

> ὅσον διαφέρουσιν οἱ φυσικῶς καὶ λογικῶς σκοποῦντες

(p. 316a, 10 f.). To be sure, the Aristotelian use of

λογικὸς

and

λογικῶς

is, thus, by no means exhausted. But we have correctly indicated the point at which a useful analysis of Aristotle's linguistic usage must, in my opinion, start. More specifically, I do not find anything in Aristotle which even remotely indicates that he understands by "logical" proof such as would operate with incorrect syllogisms and for *this* reason is not valid.

Cf. p. 200 of the *Études sur Aristote* by Ch. Thurot (Paris, 1860) which in my judgment are very instructive and still worth reading.

9. For this reason I have given in what follows the humanistically important data by tracing the history of the *nomenclature of "logic" in Aristotle's sense.* We are, therefore, dealing with an *excursus* which has in view material which to my mind must not be missing even in the most condensed outline of a history of logic and thus may be brought together under a well-defined point of view. The terms "logic," "dialectic" and so forth are always to be taken in this section as referring to the history of the nomenclature.

10. *Prantl,* I, p. 535.

11. *Prantl,* I, p. 412, 413 n. 37.

12. *De nuptiis Philologiae at Mercurii, Liber* III: *De arte Dialectica. Ed.* A. Dick (*BT,* 1925), pp. 150-210.

13. *De artibus ac disciplinis liberalium litterarum, c.* 3: *De Dialectica* (*MPL,* 70), columns 1167-1203.

14. V. Cousin: *Ouvrages inédits d'Abélard* (Paris, 1836), pp. 173-503.

15. *Thomae Aquinatis Opuscula omnia, ed.* P. Mandonnet, V: *Opuscula spuria* (Paris, 1927).

16. See below, p. 39.

17. He died in 1347 A.D. *Cf. Prantl,* III, p. 361 f.

18. *CR,* XIII (1846), p. 509 ff. *Cf.* H. Maier: *"Melanchthon als Philosoph,"* in the work *An der Grenze der Philosophie* (1909), p. 73-85.

19. *Dialectique* (Paris, 1555), the first logic of note in a modern language; *Dialecticae libri duo* (Paris, 1556).

20. *Cf.* the good and instructive analysis of M. Uedelhofen: *Die Logik Petrus Fonsecas* (Bonn, 1916), No. XIII of the collection *Renaissance und Philosophie,* edited by A. Dyroff.

21. Second edition, 1681. See below, p. 43.

22. Third edition, Sulzbach, 1685, included in *Johannis Claubergii Opera omnia philosophica, cura J. Th. Schalbruchii* (Amsterdam, 1701), p. 767 to 904.

23. Second edition, Amsterdam, 1698. Included in *Arnold Geulincx Antverpiensis Opera philosophica, rec.* J. P. N. Land, I (Haag, 1891), p. 165 to 454, with the charming sentence from the Preface which sets the reading pace for a good logic: *Ad extremum moneo, ne cursim haec legas. Euripus Logicus non patitur se navigari tam plenis velis* (*opp.* I, 173).

24. Best edition by A. Fouillée (Paris, 1878).

25. Note, *no longer Erotemata dialectices,* as Melanchthon has it still in 1547.

26. Third edition, 1692.

27. Included in the 2nd volume of the *Oeuvres* (Leiden, 1717), p. 609 to 701.

28. Fifth edition, 1729.

29. Second edition, Leipzig, 1705.

30. *Novum Organum,* 1620. Best edition by T. Fowler (Oxford, Clarendon Press, second ed., 1890).

31. In two volumes, Leipzig, 1764.

32. London, 1858.

33. *Introduction to the Doctrine of Reason,* and *Practice of the Doctrine of Reason.*

34. The exact title for the first work is: *Einleitung zu der Vernunftlehre, worinnen durch eine leichte und allen vernünftigen Menschen, waserlei Standes oder Geschlechts sie sein, verständliche Manier der Weg gezeiget wird,* ohne die Syllogistica [!] *das Wahre, Wahrscheinliche und Falsche voneinander zu entscheiden und neue Wahrheiten zu erfinden.* (Introduction to the doctrine of reason wherein the way is shown in an easy manner understandable to all reasonable human beings whatever their occupation or sex, to distinguish the true, the probable and the false and invent new truths without syllogistics.) Halle, 1691.

In the Preface, p. 16, Thomasius speaks of a *German Logic* with the title of *Kurzer Begriff der Verstand-Lehre* (Cöthen, 1621).

At all events it is, therefore, not true what it says in *ADB,* 41, p. 536, under the life of Rector Christian Weise (1642-1708) who hailed from Zittau, that his *Curieuse Fragen über die Logik* of the year 1700 (should read 1696) constitute the first German logic. Prantl has discovered a German logic even as early as 1533 in the *Bericht über die zwei ältesten Compendien der Logik in deutscher Sprache* (*AMA,* I. Cl., 1856).

The exact title for the second work is: *Ausübung der Vernunftlehre oder kurze, deutliche und wohlbegründete Handgriffe, wie man in seinem Kopfe aufräumen und sich zu Erforschung der Wahrheit geschickt machen, die erkannte Wahrheit andern beibringen, andere verstehen und auslegen, von anderer ihren Meinungen urteilen und die Irrtümer geschick-*

lich widerlegen solle. Worinnen allenthalben viel allgemeine heutzutage in Schwang gehende Irrtümer angezeiget und deutlich beantwortet werden. (Practice of the doctrine of reason, or, in brief, plain and well-established devices of how to clear up one's cobwebs and prepare oneself to explore truth, convey truth thus recognized to others, understand others and interpret them, judge other's opinions and how one should contradict errors with dexterity, through which many errors current today will be indicated and answered to the point.) Halle, 1691.

35. Halle, 1712.

36. Second edition, 1762.

37. *Discours de la méthode,* 1637, second Section. Best edition, with the detailed and very instructive commentary of E. Gilson: *René Descartes, Discours de la méthode. Texte et Commentaire* (Paris, 1925). The Cartesian criticism of Aristotle's logic may be condensed into two main statements: 1. In order to arrive at correct conclusions one does not need logic. 2. In a good logic only rules should be proposed which serve to discover *new* truths, not rules which furnish the pedantic derivation of truths long known.

With respect to the first observation, *cf.* the masterly correction by Leibniz, below, p. 52 f. But with respect to the second observation we would like to note: a) that such a logic does not exist to this very day in spite of Jacques Picard: *Essai sur la Logique de l'invention dans les sciences* (Paris, 1928); b) that its existence would imply the possibility of producing scientific geniuses, thus rendering such a logic highly improbable for any foreseeable future; c) that the Cartesian rules are so thin that one can confidently maintain that Descartes owes his magnificent accom-

plishments to *aperçus* called forth by none of these rules.

38. Best edition that of A. Fouillée (Paris, 1878). The title, *L'art de penser,* is apparently an abbreviation for a heading which ought to read something like "Instruction in the Art of Thinking." The authors themselves explain the title by *l'art de* bien penser (p. 18), so that we should say still more precisely "Instruction in the Art of Correct Thinking." *"Mais cette addition n'était pas nécessaire, étant assez marquée par le mot d'art, qui signifie de soi-même une méthode de bien faire quelque chose, comme Aristote même le remarque."* In this sense, indeed, we are dealing truly with *l'art de* penser, and not *l'art de* raisonner; for we must demand of logic *"des règles pour* toutes *les actions de l'esprit, et aussi bien pour les idées simples que pour les jugements et pour les raisonnements"* (*l. c.*).

39. Edited in 1907 by A. Buchenau on the basis of the original edition of 1701.

40. In the magnificent fragment *De l'esprit géométrique.* Best edition in the *Pensées et Opuscules,* published by L. Brunschvicg (Paris, Hachette, n.d.), p. 164-194. *Cf.* especially p. 189 ff.

41. See above, p. 11.

42. The work consists of four main parts: 1. Dianoiology, or, the theory of the laws of thought; 2. Alethiology, or, the theory of truth; 3. Semeiotics, or, the theory dealing with the names of thoughts and things; 4. Phenomenology, or, the theory of appearances.

43. It appeared in Riga.

44. Recently (1923) edited again by F. Kern, *PhB,* 72.

45. We are thinking of the famous "derivation" of the "categories" which Kant intended to furnish on the basis of the forms of judgment.

46. See above, p 4.

47. *KV*,[2] p. 187 ff.

48. *Cf.* the important and instructive section on transcendental judgment in general, *KV*,[2] 171 ff., especially p. 174: "Even though *general* logic cannot prescribe to judgment, it is quite different with *transcendental* logic, in fact, it would seem as if it were the latter's special task to correct and establish judgment by definite rules in the use of pure intellect."

49. This concept of logic in its purest form has, so far as I can see, been emphasized by the great Arabs (*Prantl,* II, p. 305-400) Al Farabi (died in 950 A.D.), to whom we owe the term "premise" (*Prantl,* II, p. 317), Avicenna (980-1037 A.D.) and Al Gazali (1059-1111 A.D.). Albertus Magnus (1193-1280 A.D.), the great teacher of Thomas Aquinas (1225-1274), adopted this concept of logic from them and transplanted it into the Latin occident. *"Argumentatio . . . Logici instrumentum, . . . Logicae . . . proprium subiectum est"* (referring expressly to the three great Arabs; *Prantl,* II, p. 310, n. 15). Likewise Thomas: *"Oportet . . . a Logica incipere . . . , quia aliae scientiae ab ipsa dependent, inquantum ipsa docet modum procedendi in omnibus scientiis"* (*Prantl,* III, p. 109, n. 489).

In this, and *only* in this sense is the designation "scholastic logic" an honorific from our point of view. Of course, it was prepared in the organological conception of Aristotelian logic by the later Peripatetics. *Cf.* the great introduction of Alexander of Aphrodisias (around 200 A.D., who was professor of

Peripatetic philosophy in Athens under Emperor Septimius Severus) to his commentary on the first book of the *Analytica priora, In Aristotelis Analyticorum priorum librum I Commentarium, ed.* M. Wallies (1883, *CAG,* II, 1), with the grand expression of dedication to the metaphysical dignity of the apparatus for gaining scientific knowledge grounded in logic (see below, p. 38).

In the 19th century it was first and foremost Herbart (see below, p. 45) who once more and vigorously breathed life into this concept.

50. The concept of a nonformal logic touched upon here covers approximately what we nowadays sometimes also call the *logic of fields. Cf.* Hans Heyse: *Der Begriff der Ganzheit und die Kantische Philosophie. Ideen zu einer regionalen Logik und Kategorienlehre* (1927). It is divided, thus, primarily into the two fields which we now designate briefly also as the "logic of the natural sciences" (Ernst Mach, Pierre Duhem, Hermann Weyl: *Philosophie der Mathematik und Naturwissenschaft,* 1926, in the *Handbuch der Philosophie* by Baeumler-Schröter, Hans Reichenbach and his antipode Hugo Dingler) and the "logic of the humanities." *Cf.* Heinrich Rickert: *Die Grenzen der naturwissenschaftlichen Begriffsbildung* (1896-1902; 5th edition, 1929), *Kulturwissenschaft und Naturwissenschaft* (1898, 6/7, 1926); Eduard Spranger: *Der Sinn der Voraussetzungslosigkeit in den Geisteswissenschaften* (*SBA,* phil.-hist. Kl., 1929, p. 2-30); Erich Rothacker: *Logik und Systematik der Geisteswissenschaften* (1926, in the *Handbuch der Philosophie* by Baeumler-Schröter), etc.

51. See p. 14.

52. Third edition, 1837. *Hauptwerke der Philoso-*

phie in originalgetreuen Neudrucken, vol. 5 (Leipzig, 1914).

53. German by J. Schiel, 1843; fourth edition, 1877.

54. See below, p. 48.

55. Fourth and fifth editions, 1920-1924.

56. Fifth edition, 1924.

57. New edition by Georg Misch (1912), *PhB,* 141.

58. *Cf.* above all the great work on the *Gegenstand der Erkenntnis* (1892, sixth edition, 1928).

59. In the case of Emil Lask (1875-1915) the material is completely available in the *Gesammelte Schriften* I (1923), II (1923), III (1924), which were edited by his pupil Eugen Herrigel, with a preface by Heinrich Rickert, his teacher. *Cf.* by all means vol. II (*"Die Logik der Philosophie und die Kategorienlehre,"* 1910; *"Die Lehre vom Urteil,"* 1911) and vol. III (*"Zum System der Logik."*).

60. I: *Die objektive Logik* (1812, second edition, 1831); II: *Die subjektive Logik oder die Lehre vom Begriff* (1816. *Works,* vol. 4 and 5 of the *Jubiläumsausgabe* of Hermann Glockner, or, vols. 3 and 4 of the edition by Georg Lasson, 1923, *PhB,* 56/57).

61. p. 15 f.

62. German and with annotations by R. Haussner, in Ostwald's *Klassiker der exakten Wissenschaften,* No's 107 and 108 (1899).

63. *Cf.* the excellent treatise by Robert Zimmermann on *"Jakob Bernoulli als Logiker"* (*SWA,* phil.-hist. Kl., vol. CVIII, 1885, p. 503-560).

64. Published in *Erkenntnis,* edited by R. Carnap and H. Reichenbach, I, 2-4 (Leipzig, 1930).

65. *Mathematische Zeitschrift,* vol. 5, p. 52-99.

66. Vol. 3 of the *Schriften zur wissenschaftlichen*

Weltauffassung, edited by Ph. Frank and M. Schlick.

67. Tornier asked me by letter *not* to mention in this connection his two relevant papers in Crelle's *Journal für die reine und angewandte Mathematik,* vol. 160 (1929) and vol. 163 (1930) since they no longer satisfied him. He was working on a textbook on the calculus of probability which has since appeared in Springer's collection *"Die Grundlagen der mathematischen Wissenschaften in Einzeldarstellungen."* Regarding the *philosophical* problems in connection with probability, Tornier is very close in his thinking to von Mises.

68. German, abridged, by F. M. Urban under title of *Ueber Wahrscheinlichkeit* (1926). The work starts in with a fine tribute to Leibniz, but in such a way that this tribute is linked with an error. "The subject matter of this book was first broached in the brain of Leibniz, who, in the dissertation, written in this twenty-third year, on the mode of electing the kings of Poland, conceived of Probability as a branch of Logic." He is referring to the *Specimen demonstrationum politicarum pro eligendo rege Polonorum, novo scribendi genere ad claram certitudinem exactum* of the year 1669 (*Opera, ed.* L. Dutens, IV, 1768, P. III, p. 522-630). Now, this youthful opus is, indeed, a highly interesting example of the fact that the mathematical method can be applied to the treatment of political problems (and that eight years before Spinoza's *Ethics!*), for it demonstrates in 60 propositions, 4 conclusions, and with reference to the fulfillment of 7 determining conditions that one would have to elect the Count Palatine Philipp Wilhelm von Neuburg as King of Poland in place of Johann Casimir who eliminated himself by resigning. But, regard-

ing the postulate of a new logic I have not even met a trace in this treatise; rather, we are dealing here basically with the application of the *old* logic to a new field, just as was the case later on with Spinoza.

However, the more emphatically we have to mention the *Specimina Juris III: Specimen certitudinis seu demonstrationum in Jure, exhibitum in doctrina Conditionum* (*WW,* VI, 1, p. 367-430) of the year 1669 which go back to the drafts of 1665. Here, on page 420, we already see the tableau of a three-valued logic in which *impossibile* receives the notation O, *contingens* ½, *necessarium* 1. It is like the very first prelude to the latest publication of the leading Polish authority on symbolic logic, Jan Lukasiewicz: *"Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalküls" (Comptes Rendus des Séances de la Société des Sciences et des Lettres de Varsovie,* XXXIII, 1930, Classe III, p. 52-77, a very interesting and considerable piece of labor. *Cf.* especially p. 65). It can also be shown that Leibniz knew—at the latest since 1678—of the two principles of the traditional probability calculus. Thus he could write justifiably March 5th, 1697, to Johann Bernoulli relative to the announced *"Ars conjectandi"* of his brother Jakob Bernoulli: *"Ego quoque talia iam olim sum meditatus" (Math.,* III, p. 377).

Cf. the nicely integrated presentation by *Couturat,* p. 239-250, in which we have to elide, however, the Polish essay adduced as a document on p. 244. In this connection the important note v on page 552 ff. in which, again, the dates 1665, or, 1672, must be substituted by 1669, and on page 553 Chapter V must be substituted by Chapter X.

69. *Gesammelte Schriften,* edited by E. Herrigel, II (1923), p. 1-282.

70. *Heidelberger Abhandlungen zur Philosophie und ihrer Geschichte,* No. 20.

71. See above, p. 1.

72. *Tim.* p. 47b:

θεὸν ἡμῖν ἀνευρεῖν

(some manuscripts read:

εὑρεῖν)

δωρήσασθαί τε ὄψιν, ἵνα τὰς ἐν οὐρανῷ τοῦ νοῦ κατιδόντες περιόδους χρησαίμεθα ἐπὶ τὰς περιφορὰς τὰς τῆς παρ' ἡμῖν διανοήσεως, συγγενεῖς ἐκείναις οὔσας, ἀταράκτοις τεταραγμένας, ἐκμαθόντες δὲ καὶ λογισμῶν, κατὰ φύσιν ὀρθότητος μετασχόντες, μιμούμενοι τὰς τοῦ θεοῦ πάντως ἀπλανεῖς οὔσας, τὰς ἐν ἡμῖν πεπλανημένας καταστησαίμεθα.

The translation which I have given I was able to discuss quite thoroughly with Herr Schöne. To him I likewise owe the important reference to the interpretation of

τεταραγμένας

by Proclus: *In Platonis Timaeum Commentaria, ed.* E. Diehl, *BT,* II (1894), p. 60, 11 ff. Proclus traces the disturbances which Plato maintains interfere in man's thought processes, back to the inhibitions residing in the very nature of the *psyche* as it gets involved in these processes as well as in its union with *soma.*

73. Why the odd and colorless title

περὶ ἑρμηνείας

has been chosen we do not know. We would expect

περὶ ἀπφάνσεως

with reference to p. 17a, 20 and 22.

74. With much regret I must confine myself in this concise treatment of logic to the above suggestion regarding the highly interesting and profound correction of Aristotelian logic by Brouwer which is quite different from the so-called Hegelian criticism and which *in this particular case* is also the logic of a Russell and a Hilbert. This suggestion at least points up sharply the decisive starting point for understanding Brouwer's standpoint and with it for any criticism worth the name.

For students of symbolic logic Brouwer's logic is now easily accessible by virtue of A. Heyting's meticulous exposition in *"Die formalen Regeln der intuitionistischen Logik"* and *"Die formalen Regeln der intuitionistischen Mathematik"* (*SBA,* phys.-math. Kl., 1930, p. 42-71 and p. 158-169). For others we recommend most of all the presentation by A. Fraenkel: *Mengenlehre*[3] (1928), paragraph 14 (with comprehensive and very reliable references to literature); furthermore the highly instructive paper on Intuitionism by Karl Menger in the *Blätter für Deutsche Philosophie,* vol. 4 (1930/31), p. 311-325.

75. *An. pr.* I, 1-22.

76. *An. pr.* I, 23 to II end.

77. I, 36.

78. I, 46.

79. II, 1-4.

80. II, 23.

81. In my judgment we cannot be sure what made Aristotle choose the title which was to all intents and purposes his according to quotations which, however, are all missing in *peri hermeneias.*

Since Plato, analytic investigations are such as serve the purpose of analysis, Platonically speaking, the

"reductio ad principia." Aristotle, likewise, says in *Eth. Nic.*, A 2, p. 1095a, 31 ff.:

μὴ λανθανέτω δ' ἡμᾶς ὅτι διαφέρουσιν οἱ ἀπὸ τῶν ἀρχῶν λόγοι

(these are the later so-called synthetic presentations)

καὶ οἱ ἐπὶ τὰς ἀρχάς

(the analytical investigations).

εὖ γὰρ καὶ Πλάτων ἠπόρει τοῦτο

(note the imperfect tense which points to a frequent discussion of these things; *cf.* Rep., VI, 511 B),

καὶ ἐζήτει πότερον ἀπὸ τῶν ἀρχῶν ἢ ἐπὶ τὰς ἀρχάς ἐστιν ἡ ὁδός, ὥσπερ ἐν τῷ σταδίῳ ἀπὸ τῶν ἀθλοθετῶν ἐπὶ τὸ πέρας ἢ ἀνάπαλιν.

According to this the analytic method is that operation which involves those processes which are needed to find for any given assertion the (axiomatic or already proven) premises from which it may be deduced. The *Analytica* of Aristotle, however, make only a very limited use of this method. The most tangible use, of course, is that of the reduction of the 12 *modi* to *"Barbara"* and *"Celarent."* We should, perhaps, translate better: "Investigations toward establishing a foundation for gaining scientific knowledge," or "Foundations of acquiring scientific knowledge," or still more anemically: "Contributions to fundamental research."

Kant's interpretation which justifies the analytic character of Aristotelian logic by saying that it "dissolves the entire formal business of the understanding and reason into its elements" (*KV*,[2] 84), is obviously insufficient also in this limitation, for he does not define what we are to understand by this reduction. He confines himself basically to the *Analytica priora* which is historically, of course, not permissible.

82. *C.* 10.

83. *C.* 13.

84. *C.* 4, 8, 9.

85. Occasionally also other maxims appear, for instance the maxim which we may still heed, perhaps today more than ever before: Designate things by the names that everyone gives them, but you need not judge them as everyone does. *Top.* II, 2, p. 110a, 16 ff.:

> ταῖς μὲν ὀνομασίαις τὰ πράγματα προσαγορευτέον καθάπερ οἱ πολλοί, ποῖα δὲ τῶν πραγμάτων ἐστὶ τοιαῦτα ἢ τοιαῦτα, οὐκέτι προσεκτέον τοῖς πολλοῖς.

The interpretation just given furnishes also the most necessary material for expressing Kant's interpretation of "topic" in more precise terms: "We may call every concept, every title, by which we comprehend a whole lot of knowledge [note: what is meant by that?], a *logical area.* It is on this that *Aristotle's logical Topics* are based. Teachers and rhetoricians were to make use of them in order to see, when certain captions for thought are given, what would best suit the matter on hand and then proceed to rationalize with an air of thoroughness or prattle in so many words" (*KV,*[2] 324 f.).

86. More in the Bibliographic Appendix.

87. To make headway, the *Topica* may be skipped.

88. Our sketch agrees essentially with the picture projected by Friedrich Solmsen in 1929 in his comprehensive investigation of the *Entwicklung der Aristotelischen Logik und Rhetorik. Cf.* p. 37 f.: "The dialectic and the apodeictic, *i.e.,* scientific syllogisms, were in existence early in Aristotle's time, long before

syllogistics had been developed. They were independent methods, absolutely apart from general syllogistics as developed by Aristotle in the *Analytica priora* . . . We should not . . . look upon apodeictic and dialectic merely as two fields of application and forms in which the syllogistic process appears in the *Analytica priora* when thoroughly checked as to all its modalities." To be sure, Solmsen is justified in adding that, considering the form in which we meet it, the apodeictic theory of Aristotle is most intimately connected internally as well as externally, with the general syllogistic theory of the *Analytica priora* (p. 38). Regarding the *Sophistici Elenchi* he maintains (p. 70) that they were probably composed somewhat later than the rest of the books of the *Topica* which plainly belong in the period of the Academy. I would place them much later into a period in which Aristotle had already found the essential results of syllogistics. The opinion which wants to elucidate the *topoi* as points of view, methods, aids or categories, I think is exaggerated. They are supposed to be not only off the track and pointless but directly responsible for creating mischief (p. 165). I gladly concede this for the "categories" and even for the "methods," but not for the "points of view" and "aids." I may be all the more justified in holding on to this interpretation as Solmsen likewise, and in spite of persisting efforts, has not succeeded so far as I can see in stating even approximately and precisely what, after all, the *topoi* of Aristotle really are. His interpretation of the concept of *Analytica* (p. 123, note 2) I furthermore do not hold to be convincing. Finally, I have not been able to see in how far the *Analytica* should not only be later than the *Dialectic,* but nullify it (p. 26).

89. *Top.* I, 1, p. 100a, 25 f. *Cf.* the additional remarks in *An. pr.* I, 1, p. 20 ff.

90. See below, p. 45 ff.

91. *WL,* II (1837), p. 200.

92. *Cf.* the work of Friedrich Weidauer: *Zur Syllogistik* (1928; *Neue Psychologische Studien,* edited by Felix Krueger, III, 4), p. 596-626, which is most instructive in this connection, but only in this connection. The Babel of languages is a rather harmless thing in comparison with the chaos on which Weidauer reports. For such a logic even Descartes' criticism in the famous preface to the French translation of the *Principia Philosophiae* is almost too lenient: *"La logique . . . corrompt le bon sens plutôt qu'elle ne l'augmente"* (*AT,* IX,^{2} 13, 24 and 29 f.).

93. *An. pr.* I, 1, p. 24b, 28 ff.

94. *Von der falschen Spitzfindigkeit der vier syllogistischen Figuren* (1762), paragraph 2, *PhB,* 46a (1905), p. 57.

95. In the logical, *not* the psychological-pedagogical sense!

96. See above, p. 3-4.

97. The first to state it clearly was Jan Lukasiewicz in the treatise quoted above on p. 102. May I say that this example has lent me moral support in what I have to say below in exoneration of the Stoics in the face of all "good" tradition. It needs be said all the more because both chapters on the logic of the Stoics in Victor Brochard: *Études de philosophie ancienne et de philosophie moderne* (Paris, 1926), p. 221-251 also do not mention a word of it. There is no reference to it either in Emile Bréhier: *Histoire de la philosophie,* I, 2 (Paris 1927), p. 300 ff.

Philosophical critics who would be inclined to

look for the bases of the traditional contempt for Stoical logic even "deeper," may be expressly reminded that as early as the Stoics we meet the same "frightening" entanglement of an extreme formalism in logic with an equally extreme positivism in epistemology as is characteristic for the "Vienna Circle" (see the Bibliographic Appendix).

98. The best collection of fragments has been made by Hans v. Arnim: *Stoicorum veterum fragmenta* (Leipzig, Teubner, 1902 ff.; vols. I-III in photoelectric reprint 1921 ff., vol. IV, 1924, containing the detailed indices compiled by Maximilian Adler). For Chrysippus' logic *cf.,* above all, vol. II, n. 45-298 of this collection.

For the Stoics, Lukasiewicz (see above, note 68), p. 67, note 17, extracted the syllogistic rule

εἰ οὐ τὸ πρῶτον, τὸ πρῶτον· τὸ πρῶτον ἄρα,

which is highly interesting for the student of symbolic logic, from Sextus Empiricus: *Adv. Math.,* VIII, 292 *(Sexti Empirici opera, rec.* H. Mutschmann, *BT,* II, 1914, p. 170, 30 ff.).

We cannot locate this passage in v. Arnim so that we have to reckon with the possibility that the material on the logic of the Stoics collected here is also in other respects not wholly satisfactory.

In this case also I have quoted as much as possible *Prantl* in order to facilitate for the reader the consulting of sources which is necessary for a beginning orientation, and at the same time stimulate him to compare our view with the destructive criticism that one meets again and again in *Prantl.*

We can only allude to the highly interesting history of the peculiar inferential schematism: "If p is false, p is true; therefore p is true," which begins

with *Euclid* IX, 12, and its counterpart: "If p is true, p is false; therefore p is false," which is attested already in Plato's *Theaetetus* (p. 171 A). The elements of this history may be looked up in Bolzano's *Wissenschaftslehre,* vol. 4, paragraph 530, note 3, p. 280 ff. and quite independently of this in G. Vailati: *A proposito d'un passo del Teetelo e di una dimostrazione di Euclide (Scritti,* Leipzig-Firenze, 1911, p. 516 to 527).

99. *Prantl,* I, p. 471 ff.

100. *Prantl,* I, p. 375 ff.

101. *Prantl,* I, p. 385.

102. *Prantl,* I, p. 368.

103. Cicero, *de fato,* 10, 21 (*M. Tullii Ciceronis scripta quae manserunt omnia, rec.* C. F. W. Mueller, IV, 2, Leipzig, 1890, p. 259, 37 ff.): *Hic primum si mihi libeat adsentiri Epicuro et negare omnem enuntiationem aut veram esse aut falsam, eam plagam potius accipiam quam fato omnia fieri conprobem; illa enim sententia habet aliquid disputationis, haec vero est non tolerabilis. Itaque contendit omnes nervos Chrysippus, ut persuadeat omne* axioma *aut verum esse aut falsum. Ut enim Epicurus veretur, ne si hoc concesserit, concedendum sit fato fieri quaecumque fiant . . . , sic Chrysippus metuit, ne, si non obtinuerit omne, quod enuntietur, aut verum esse aut falsum, non teneat omnia fato fieri et ex causis aeternis rerum futurarum.*

104. See above, p. 25.

105. *Prantl,* I, p. 438, note 109 and I, p. 450, note 136. *Cf.* above, note 103.

106. *Prantl,* I, p. 443, note 118.

107. *Prantl,* I, p. 454.

108. *Prantl,* I, p. 471 ff.

109. We are thinking of the postulate *palpabilia veritatis criteria* formulated again and again and justly by Leibniz against Descartes and even against the magnificent axiomatic of Pascal (*B,* 82). The Stoical table of values furnishes such a criterion for checking any and all if-so assertions, in fact, the only exact criterion known today. We should compare with this, *e.g.,* the sterility of the voluminous discussions of the hypothetical judgment in Benno Erdmann's *Logik* [3] (1923), p. 523-550, with the traditional criticism of the Stoics and the disjunction between the theories of the conclusion and the consequent in a hypothetical judgment so totally unsatisfactory for a thinker who makes rigorous demands.

110. I suspect that for *this* reason and in contrast with the Peripatetics the Stoics considered logic (called dialectic by them) an integrating constituent of philosophy and rejected the organological view of the Peripatetics.

111. *Cf.* the splendid explanation in *In Aristotelis Analyticorum priorum librum I Commentarium, ed.* M. Wallies, *CAG,* II, 1 (1883), p. 4, 30 f.:

> οὐκ ἐπεὶ δὲ ὄργανον καὶ οὐ μέρος φιλοσοφίας ἡ ἀναλυτική, διὰ τοῦτο ἐλάττονός ἐστιν ἡμῖν σπουδῆς ἀξία.

Proof p. 6, 8 ff.:

> εἰ (δὴ) τὸ θεῷ ὁμοιοῦσθαι

(cf. Plato, *Theaet.,* p. 176B)

> μέγιστον ἀγαθὸν ἀνθρώπῳ, τοῦτο δ' αὐτῷ διὰ δεωρίας τε καὶ τῆς τἀληθοῦς γνώσεως περιγίνεται, ἡ δὲ τἀληθοῦς γνῶσις δι' ἀποδείξεως, δικαίως ἂν πλείστης τιμῆς ἀξιοῖτο καὶ σπουδῆς, διὰ δ' αὐτὴν καὶ ἡ συλλογιστική, εἴ γε ἡ ἀπόδειξις συλλογισμός τις.

This proof is not only an example of the fact that there were great men in the age of the "declining" ancient world, even in professorial ranks, but an invaluable testimonial for the unparalleled zeal of Platonism. By such men we can recognize the stature of Plato and form an approximate idea even without having read a line of Plato of what western man owes him and, of course, Aristotle whom we must name in closest connection with him. For Alexander's syllogistics cf. the study of G. Volait: *Die Stellung des Alexander von Aphrodisias zur Aristotelischen Schlusslehre* (1907; *APhG,* XXVII).

112. *De propriis libris c.* 11 (*Claudii Galeni Pergameni scripta minora* II, *rec.* J. Mueller, *BT,* 1891, p. 116, 20 ff.):

> νὴ τοὺς Θεούς, ὅσον ἐπὶ τοῖς διδασκάλοις, εἰς τὴν τῶν Πυρρωνείων ἀπορίαν ἐνεπεπτώκειν ἂν καὶ αὐτός, εἰ μὴ καὶ τὰ κατὰ γεωμετρίαν ἀριθμητικήν τε καὶ λογιστικὴν κατεῖχον, ἐν αἷς ἐπὶ πλεῖστον ὑπὸ τῷ πατρὶ παιδευόμενος ἐξ ἀρχῆς προεληλύθειν ἀπὸ πάππου τε καὶ προπάππου διαδεδεγμένῳ τὴν θεωρίαν. ὁρῶν οὖν οὐ μόνον ἐναργῶς ἀληθῆ φαινόμενά μοι τὰ κατὰ τὰς ἐκλείψεων προρρήσεις . . ., βέλτιον ὠήθην εἶναι τῷ τύπῳ τῶν γεωμετρικῶν ἀποδείξεων χρῆσθαι

(unfortunately the text has been corrupted exactly at the place which is critical for us, probably by a mechanical copyist, to

τὸν τύπον

—*Prantl,* I, p. 562, note 81 has even

> τόπον — τὸν γεωμετρικὸν ἀποδείξει χρῆσθαι). Καὶ γὰρ καὶ αὐτοὺς τοὺς διαλεκτικωτάτους καὶ φιλοσόφους οὐ μόνον ἀλλήλοις ἀλλὰ καὶ ἑαυτοῖς ηὕρισκον διαφερομένους ἐπαινοῦντας

ὅμως ἅπαντας ὡσαύτως τὰς γεωμετρικὰς ἀποδείξεις.

For us, Galen is also the first who speaks of

λογικαὶ ἀρχαί

in the plural. He did so with reference to the axiomatization of logic which he had in mind. But *Prantl,* I, p. 563 declares this as nonsense which the stupidity of formal logic characteristically never repudiated, and justifies the remark on the ground that a unified discipline can as a matter of course rest only on One principle. It is truly disturbing that a historian of logic could have written that, especially around the middle of the 19th century.

On Galen as a logician one should above all consult the neat work of K. Kalbfleisch on Galen's introduction to logic (*Jahrbuch für klassische Phil.,* vol. 23, supplementary volume, 1897, p. 679-708); furthermore the illuminating treatise by J. v. Mueller on Galen's work concerning scientific proof (*AMA,* I Cl., 1895, p. 405-478).

As to the traditional first name, Claudius, I owe Hermann Schöne the information that this alleged first name is not attested prior to the Renaissance and probably arose in an erroneous explanation of Cl. Galenus, as handed down in Latin manuscripts, as Claudius instead of Clarissimus Galenus.

113. See above, p. 5-6.

114. The *Logica demonstrativa* had its first edition probably in 1692, other editions came out in 1697 and 1702. The reprint I have used is *Augustae Ubiorum* (Köln, 1735).

Cf. in this connection the fine treatise by G. Vailati: *Di un'opera dimenticata del P. Gerolamo Saccheri* (*"Logica demonstrativa,"* 1697) in *Scritti* (Leip-

zig-Firenze, 1911, p. 477-484). Despite all efforts Vailati could not obtain a copy of the edition I have used. He based his work on an anonymous copy of the year 1697 which is kept at the Brera of Milano. This is probably the only one which is in existence apart from the one at Münster. For this reason alone it would be highly desirable to reprint this little book in small-octavo comprising only 162 pages. An even more cogent reason is the fact that so far as content is concerned, it stands high above anything that we can gather nowadays for this type of logic from the best compendia of like volumes in the field of qualitative logic.

Regarding the date let us observe that the reprint I used knows neither the first edition of 1692 mentioned by the Saccheri biographer Gambarana (*cf.* Engel und Stäckel: *Die Theorie der Parallellinien von Euklid bis auf Gauss,* 1895, p. 34 f.), nor the edition of 1697 discovered by Vailati, but only an edition of 1702 which appeared at Ticini (at Pavia where Saccheri taught since 1697). The latter did not appear in 1701 as Vailati states (p. 478) after Gambarana's unpublished biography of Saccheri. The year in which Saccheri was born is apparently not 1662 (Vailati *op. cit.,* p. 522), but 1667.

115. *De syllogismo hypothetico libri duo, MPL,* 54, col. 831 ff.

116. *Cf.* above all the *opusculum de divina omnipotentia* (*MPL* 145, col. 595-622), *c.* 11: *Quod naturae conditor naturae sit etiam immutator.* Col. 612 we meet the famous sentence: *Haec impossibilitas* (*sc.* of the existence of a thing with contradictory properties) *recte quidem dicitur, si ad naturae referatur inopiam: absit autem, ut ad maiestatem sit applicanda divinam.*

To my knowledge the history of this theological antilogism has not yet been written. Thomas, so far as I can see in my researches, has corrected it quietly by attempting to differentiate the suprarational from the absurd. One of Kepler's remarks on how it is with these things, which he made at the beginning of the 17th century, is instructive. He says in *Harmonice Mundi I* (*Opera, ed.* Frisch, vol. V, p. 407, note): *Nihil est vulgatius apud theologos quam . . . Dei scientiam ad . . . impossibilia se non extendere.*

Worth reading even now is F. J. Clemens: *De Scholasticorum sententia "philosophiam esse theologiae ancillam" commentatio* (Münster i. W., n.d.).

117. *Prantl,* III, p. 32 to 74.

118. *Cf. Ueberweg-Geyer,* paragraphs 27 and 37.

119. *WL,* II, paragraphs 169-184.

120. German translation by Gumbel and Gordon under the title of *Einführung in die mathematische Philosophie* (1923; 2nd ed., 1931).

121. *Dialectices libri duo,* 1556.

122. They are mentioned neither by Ch. Waddington in the rather voluminous but, for Ramist logic quite insufficient work on *Ramus, sa vie, ses écrits et ses opinions* (Paris, 1855), nor in the treatise by Prantl in *SMA,* phil.-hist. Kl., 1878, vol. 2, p. 159-169.

123. *Prantl,* III, p. 142, note 624 and IV, p. 231, note 206.

124. *Prantl,* III, p. 400.

125. *CR,* XIII (1846), col. 611 f.

126. Fifth edition, 1729, p. 184 ff.

127. Fifth edition, 1729, p. 261-279.

128. This "proof" is unsupportable because it operates with the logical extension of *individuals,*

that is, with the "extension" of entities for which "extension" has not been defined at all. Up to now we have made a beginning only with the definition of the logical extension of a *property* by virtue of the totality of the individuals *possessing* this property. More we cannot say at this point. Unfortunately, I cannot go into details of the dismal but not uninteresting history of the "classical" theory of the extension of concepts and the theory of the intension of concepts which is intimately connected with it. For the present stand of this "theory" it will pay to study the chapter on extension (*Umfang*) in B. Erdmann's *Logik* [3] (1923), paragraphs 173-202. Erdmann, too, subscribes still to the mistaken theory of the so-called reciprocity of the concept of *extension* and the concept of *intension,* despite the criticism which already Bolzano, *WL,* I, paragraph 120, loosed on this doctrine. What should take its place has been indicated by Walter Dubislav in paragraph 63 of his informative monograph on definition, which appeared in third edition in 1931.

129. II, 3, p. 119 (Fouillée).

130. *Lettres à une princesse d'Allemagne,* which came out in 1770 (Mitau and Leipzig), Nr. CVII.

131. Third edition, 1870.

132. See above, p. 13.

133. See above, p. 11.

134. I, 6, p. 54 (Fouillée).

135. Hamburg, 1638; second edition prepared by Joh. Vagetius who contributed to this work a very informative appendix on the merits of the *Logica Hamburgensis* especially in the field of the "oblique" syllogisms (see above, p. 26). This edition also contains a keen criticism of the attempted innovations in

the *Logic of Port Royal,* III, 10 and 11, toward a symbolic logic (III, 10: *Principe général par lequel, sans aucune réduction aux figures et aux modes, on peut juger de la bonté ou du défaut de tout syllogisme).*

136. *Cf.* the splendid article by Ernst Cassirer: *"Leibniz und Jungius"* in *Beiträgen zur Jungius-Forschung, Prolegomena zu der von der Hamburgischen Universität beschlossenen Ausgabe der Werke von Joachim Jungius* (Hamburg, 1929).

137. P. 137. The equipollence was already noted by Galen *(Prantl,* I, p. 606).

138. P. 180.

139. *De interpret., c.* 11, p. 20b, 31 to p. 21a, 33.

140. P. 181.

141. See above, p. 26.

142. See above, p. 11-12.

143. See above, p. 39.

144. *Dissertatio de usu et limitibus principii rationis determinantis vulgo sufficientis* (Leipzig, 1743), in the *Opuscula philosophico-theologica* (Leipzig, 1750), p. 152-294, German by Chr. Fr. Pezold (Leipzig, 1766): *Herrn S. Christian August Crusii Ausführliche Abhandlung von dem rechten Gebrauche und der Einschränkung des sogenannten Satzes vom zureichenden oder besser determinierenden Grunde.* On p. 41 ff. of this treatise the deduction of the Principle of Sufficient Reason attempted by Wolff from the axiom of the excluded contradiction (in paragraph 30 of the German *Metaphysik* and paragraph 70 of the *Ontologia*)—which rests on the amazing minor premise that Nothing cannot be a reason, so that for *that* reason everything must have a reason—is subjected to a criticism which is devastating because it proceeds

with the infallibility of instinct, but not quite thoroughgoingly enough. Let us ask ourselves, which qualitative logic has up to now really and finally done away with this attempted deduction? In the metaphysics of Catholicism this problem is still hotly disputed. *Cf.* J. Geyser: *Das Prinzip vom zureichenden Grunde. Eine logisch-ontologische Untersuchung* (1929). I cannot say more in this place about the rather singular and highly profound function of this principle in Leibniz' metaphysics, a function which is most intimately bound up with his interpretation of logic as the theory of the condition of existence of all possible and *only* possible worlds.

145. See above, p. 12.

146. In the famous *Lettres à une princesse d'Allemagne,* II (1770), Nos. CII-CV, of February, 1661.

147. Let it be mentioned here that already Leibniz operated with these circles of Euler—and with how many other for the most part highly ingenious symbolizations besides! *Cf.* the instructive references in *Couturat,* p. 21-32 and p. 113 ff.

The symbolization by means of figures of the conceptual relationship entering syllogistic rules as such reaches back much farther. Already Julius Pacius makes very extensive use of this symbolization in his 1584 edition of the *Organon* with commentary (see the Bibliographic Appendix) which is still quite valuable. He does so with a matter-of-factness from which we may conclude that he had predecessors. I have come across circles in literature first in the very interesting little book of Johann Christoph Sturm (1635-1703): *Universalia Euclidea* (Haag, 1661), the mathematician from Altorf whom Leibniz also esteemed highly. In this booklet 35 propositions of Euclid's

proportional theory are reduced to 17 and expounded with essentially simplified proofs which, of course, in this precarious field need thorough scrutiny. They are followed in an appendix by the *Novi Syllogizandi Modi* to which the predicate "novel" applies only in so far as they are rather uninteresting, to be sure, but at all events correct and not to be met with in Aristotelian logic. It is here, on page 86 ff., that frequent use is made of circles in order to highlight the evidence, *not* to lighten the responsibility of proving his point. These circles were employed methodically in literature for visualizing the *whole* of the Aristotelian syllogistic probably for the first time in the very interesting revision of the *Nucleus Logicae* (Leipzig, 1691), a most insignificant booklet of 72 pages by Christian Weise (1642-1708), a Rector from Zittau. The revision, which was dedicated to the Berlin Academy and hence to Leibniz, was done by Joh. Christian Lange (1669-1756), Professor of Philosophy at Giessen, later Superintendent of Schools and Churches in Hessia, a most formidable logician to whose almost total oblivion we cannot assign any sufficient reason. The revision appeared under the title of *Nucleus Logicae Weisianae . . . sic auctus et illustratus, ut vera ac solida Logicae peripatetico scholasticae purioris fundamenta detegantur et ratione mathematica per varias schematicas praefigurationes huic usui inservientes ad ocularem evidentiam deducta proponantur* (Giessen, 1712). The booklet of 72 pages became in the revised state an opus of nearly 850 pages. The detailed *schematica illationis syllogisticae delineatio* may be consulted on pages 249-345, with interesting observations concerning the predecessors on page 248.

What Ziehen: *Lehrbuch der Logik* (1920), p. 228

says concerning the use of geometrical symbols by Johannes Philoponos (6th century A.D.) is just as apocryphal as the endeavor among other historians of logic to trace Euler's circles back to Christian Weise.

148. He is referring to the prototype of mathematics which has, however, by no means been attained.

149. *PhB,* 46a.

150. Paragraphs 34-80.

151. Fourth edition, 1837, re-edited by K. Häntsch, *PhB,* 146 (1912).

152. Fifth edition, 1887.

153. Reprints at Felix Meiner, Leipzig, 1929-31.

154. See above, p. 3-4.

155. *Cf.* especially *WL,* II, paragraph 154 ff., 198ff.; I, paragraph 95 ff.

156. See above, p. 40.

157. *Cf.* the little book of Bolzano's keenly critical pupil which I brought out in 1931 in a new edition together with W. Dubislav and which appeared in the Felix Meiner Verlag in Leipzig. It is F. Prikonsky: *Neuer Anti-Kant oder Prüfung der Kritik der reinen Vernunft nach den in Bolzanos Wissenschaftslehre niedergelegten Begriffen.* Here we also get acquainted with Augustine's anticipation of Bolzano's principles and ideas, a fact hardly commented on to this day.

158. The fourth edition of vol. I and vol. II, 1 appeared in 1928, the second edition of vol. II, 2 appeared in 1921.

159. Third edition of 1923 prepared by Erich Becher.

160. Fourth edition, 1906; reprinted in 1928.

161. Provided we keep them at a sufficient distance from each other we could name besides the

work of Keynes the splendid work of Tilman Pesch, S. J. (1836-1899): *Institutiones logicae* [2] (1914), edited by C. Frick, S. J., as one of the best presentations of scholastically interpreted formal logic.

162. The works on logic by Sigwart and Wundt referred to above, p. 18, cannot be critically treated here since their main concern is throughout with the field of methodology. Similarly with Lotze's *Logik* whose critique belongs rather in a history of the philosophy of value and Plato-interpretation. As to content, Hermann Cohen (1842-1918): *Logik der reinen Erkenntnis* (1902; third edition, 1922) also belongs into the last third of the 19th century. A discussion of it does not fit in here at all but belongs into the history of Neo-Kantianism.

163. Suggested in the *Psychologie* of 1874 and carried out by F. Hillebrand: *Die neuen Theorien der kategorischen Schlüsse* (1891).

164. Let us mention at least among the writings of Alexius Meinong (1853-1920) his voluminous monograph *Ueber Annahmen* (1901; second edition, 1910). His philosophic legacy, stimulating, conceived in serious labor and hence very individual in character, should at any rate *not* be subjected to criticism here as to its yield in formal logic which, after all, we are concerned with here. Paragraph 31 on the nature of hypothetical judgment, of course, appears to me to be only a new and saddening proof of the insurmountable sterility of a theory of judgment which is not primarily oriented on the problem of the criterion of truth and the demands of the syllogism.

165. Part II appeared in 1922, Part III in 1924.

166. Second edition, 1929.

167. See above, p. 39.

168. *Phil.,* VII, p. 514-527.

169. P. 516.

170. *La logique de Port Royal,* III, Introduction, p. 174 (Fouillée): *La plupart des erreurs des hommes . . . viennent bien plus de ce qu'ils raisonnent sur de faux principes, que non pas de ce qu'ils raisonnent mal suivant leur principes.*

Even today nearly all philosophers think this way still in spite of Leibniz—at any rate those among them who are critics of symbolic logic.

171. P. 519.

172. P. 525.

173. P. 523.

174. P. 519 f.

175. *Cf.* the splendid remarks in the manifesto of 1694 *"De primae philosophiae emendatione"* (*Phil.,* IV, p. 468 ff.): *Mihi vero in his* [*sc. Metaphysicis*] *magis quam in Mathematicis luce et certitudine opus videtur . . . Itaque peculiaris quaedam proponendi ratio necessaria est, et velut filum in Labyrintho, cuius ope non minus quam Euclidea methodo ad calculi instar quaestiones resolvantur* (p. 469).

Cf. NE, IV, 2 (*Phil.,* V, p. 351 f.; German *PhB,* 69 [3] (1926), p. 430 ff.) and *Phil.,* VII, 323 f.

176. *Calculus ratiocinator seu artificium facile et infallibiliter ratiocinandi. Res hactenus ignota* (*Fr.* 239).

Cf. the *"Fundamenta calculi ratiocinatoris," Phil.,* VII, 204 ff.

177. In 1678, writing to Tschirnhausen (*Math.,* IV, 462): *"Nihil aliud enim est calculus quam operatio per characteres, quae* non solum in quantitatibus, sed in omni alia ratiocinatione *locum habet."*

"Non omnes formulae significant quantitatem, et infiniti modi calculandi excogitari possunt" (*Fr.* 556).

In the illuminating treatise *Mathesis universalis* (*Math.*, VII, p. 49-76) the new logic, the Logica Mathematica sive Mathesis universalis sive Logistica(!) sive Logica Mathematicorum (p. 54) is, as *scientia generalis de* qualitate expressly contrasted with mathematics in the ordinary sense, that is, the *Mathesis specialis* as the *scientia generalis de* quantitate (p. 61). By *Logica Mathematicorum* in Leibniz' sense we have to understand a logic that can only be created *by* a mathematician and by no means a logic which needs to be of interest *only* to mathematicians.

Very instructive is, finally, No. XVIII of *Math.*, VII (p. 203-216). Here we find that the later "Algebra of Logic" (G. Boole, A. de Morgan, E. Schröder; see below, p. 58) is rejected beforehand with the almost incomprehensible unfailingness of an instinct. The objection which could be raised at least with *some* justification against the "Algebra of Logic" that it was quantifying, has long ago become exceedingly obtuse and did not even exist for Leibniz. *"Errant . . . qui ab ea* [*sc. ab Algebra*] *quidvis sibi pollicentur et de viribus eius sentiunt immoderati et* [*eam*] *pro arte inveniendi atque analysi in universum ac scientiarum principe habent"* (p. 203). It is expressly stated further: "Calculus in universum . . . longissime distat ab Algebra . . . *Dantur enim Calculi quidam ab hactenus usitatis plane diversi, ubi notae sive characteres* non quantitates *sive numeros definitos vel indefinitos, sed alias plane res, verbi gratia puncta,* qualitates, *respectus significant"* (p. 207).

If, at last, we add that our mathematics was, for

Leibniz, the result of analytical judgments, hence a branch of logic, then *the new logic, instead of furnishing a disreputable quantification of qualities, furnishes rather a magnificent qualification of quantities.* At least we had to touch upon this ultimate consequence which Leibniz, let it be noted, had not yet drawn explicitly. For, if it is correct, and since Frege (see below, p. 59) holds true, indeed, in a very profound sense, then it completely reverses the common conception. Now, this is exactly and fully what this antiquated conception deserves.

178. I cannot go into details with respect to Leibniz' profound concept of representation which has its origin, of course, in the pictorial representation of equations by curves and *vice versa.*

179. The new logic thus is based on nothing less than a system of "meaningless" symbols. This continually recurring yet absolutely erroneous assertion is immediately done away with if we can point out that merely when we *operate* with these symbols—this we will have to do, of course—we are arranging it so that *during the operation* with these symbols we need not think of their meaning. Indeed, it would be best if we generally did abstract from the meaning so as to be sure that nothing has crept in that rests solely on considerations of content.

180. Immediately the famous principle of Heinrich Hertz from the Foreword to his *Mechanik* comes to mind. *Cf.,* for instance, W. Dubislav: *Die Definition*[3] (1931), paragraph 51. The three postulates of Leibniz constitute the best anticipation of this principle I know of. Perhaps they are more than merely anticipatory.

181. Especially in *Phil.,* VII and the "Fragments." Consult further *NE,* IV (*Phil.,* V). *Cf.* the masterly presentation of Couturat's.

182. *B,* p. 80 f.

183. *B,* p. 97. We cannot discuss in detail how far Leibniz was with these remarks in advance of the famous *"Ars magna"* of Raymundus Lullus (1235-1315). *Cf. Prantl,* III, p. 145-177; *Ueberweg-Geyer,* p. 459 f.; C. Ottaviano: *L'ars compendiosa de R. Lulle. Avec une étude sur la bibliographie et le fond Ambrosien de Lulle* (Paris, 1930; *Études de philosophie médiévale,* Director E. Gilson, XII). At any rate, there are *also* sentences in the program of this peculiar man which play surprisingly over into Leibniz, tentatively at least. Again, the Leibnizian program which the text unfolds, must likewise be first extracted from a much more comprehensive material which is not always fully perspicacious in detail. Let us by all means emphasize this here. Without Russell's symbolic logic it would not have been possible for me to present this program as I have done in the text. Whoever feels that this is a blemish operates with another standard of the true and good which naturally can be "contradicted" just as little as any other concept, but may only be accepted or rejected. The purpose that I had in mind and felt competent to carry out was that I had to report not *on* Leibniz in general, but on Leibniz as *the creator of symbolic logic.* And I have no objection if anyone will consider it just a preliminary answer to the problem in how far Leibniz is to be looked upon as the creator of symbolic logic.

184. *Fr.* 420.

185. *Phil.,* VII, p. 200. The reader who has fol-

lowed thus far will now be able to judge for himself the "criticism" which Hegel made in his *Logik* of this man Leibniz. The logic of Leibniz is, for Hegel—who, it must be owned, got a bit more acquainted with Leibniz' logic than do most Hegelians, by virtue of his teacher Ploucquet—a "pet idea of Leibniz which he conceived in his youth and did not give up even later in life despite its immaturity and shallowness" (*WW*, V, p. 147).

186. *KV*,[2] p. 740 ff.

187. *NE*, vol. IV, p. 12; *Phil.*, V, p. 431 ff.

188. *"Il faut ajouter que même des principes, dont la certitude n'est pas entière, peuvent avoir leur usage, si l'on ne bâtit là dessus que par démonstration. Car quoique* toutes les conclusions en ce cas ne soient que conditionelles et vaillent seulement en supposant que ce principe est vrai, *néanmoins cette liaison même et ces énonciations conditionelles seraient au moins démontrées; de sorte* qu'il serait fort à souhaiter que nous eussions beaucoup de livres écrits de cette manière, *où il n'y aurait aucun danger d'erreur, le lecteur ou disciple étant averti de la condition"* (V, 431).

189. Second edition of vol. I, 1925; of vol. II and III, 1927.

190. On Lambert as a symbolic logician we have to mention primarily the treatise *"De universaliori Calculi idea"* (1765) which toys with Leibniz' calculus of qualities, in the *Acta Eruditorum* (1764/65, p. 441 to 473); furthermore the relevant material in the *Logischen und philosophischen Abhandlungen* (Berlin, I, 1782; II, 1787) which were prepared for print by Joh. Bernoulli, and in vols. 1 and 2 of the correspondence likewise edited by Joh. Bernoulli: *Joh.*

Heinrich Lamberts deutscher gelehrter Briefwechsel, 5 vols. (Berlin, 1781-85).

The best exposition of his achievements in the form of a condensed survey is in the excellent work by John Venn: *Symbolic Logic* [2] (London 1894), p. xxxi-xxxvi. Much thinner is the detailed review by G. Stammler: *Begriff, Urteil, Schluss; Untersuchungen über Grundlagen und Aufbau der Logik* (1928), p. 83-119.

The main source for Ploucquet is the *Sammlung der Schriften, welche den logischen Kalkül Herrn Prof. Ploucquets betreffen, mit neuen Zusätzen* (Tübingen, 1773; copy in the library of the University of Kiel). Furthermore, the able treatise by Karl Aner: *Gottfried Ploucquets Leben und Lehren* (*APhG,* vol. XXXIII, 1909).

191. Main work: *Formal Logic or the Calculus of Inference, Necessary and Probable* (London, 1847). New edition by A. E. Taylor, London, 1926.

192. Principal work: *An Investigation of the Laws of Thought, on which are founded the mathematical theories of Logic and Probabilities* (London, 1854). New edition by Philip E. B. Jourdain, London, 1916.

It should be noted that both works intend to correct at the same time the principles and methods of the calculus of probability.

193. *Vorlesungen über die Algebra der Logik* (I, 1890; II, 1, 1891; III, 1895; II, 2 ed. by E. Müller, 1905).

An excellent précis of this "algebra" which, strictly speaking, has only a historic significance today and may *not* be used as authority for judging symbolic logic, has been written by Louis Couturat: *L'Algèbre de la Logique* (Paris, 1905; second edition, 1914).

194. Main works: *Formulaire de Mathématiques* (Paris, 1901); *Formulario mathematico,* ed. V. Torino (1908).

195. Principal works: *Die Grundlagen der Arithmetik* (1884); *Grundgesetze der Arithmetik* (I, 1893; II, 1903).

196. *NE,* IV, 6; *Phil.,* V, p. 379. Already in the *Dissertatio de arte combinatoria* did Leibniz call in 1666 for *notas quam maxime naturales* (*Phil.,* IV, p. 73; *WW,* VI, 1, p. 202, 5).

197. See above, p. 3-4.

198. See above, note 74.

199. See above, p. 41.

200. See above, p. 43 f.

201. *Principia Mathematica,* I, *37'2.

202. See above, p. 31.

203. Think of the devastating consequences which the inexact concept of syllogizing has had in the interpretation of Descartes' *Cogito, ergo sum! Cf.* my paper on *"Cogito, ergo sum"* in *Kantstudien,* XXXVI (1931).

204. For a preliminary orientation see, for instance, the very good review of the acceptable theories of judgment by J. K. Kreibig: *Die intellektuellen Functionen, Untersuchungen über die Grenzfragen der Logik, Psychologie und Erkenntnistheorie* (1909), p. 183-200, which has not been superseded. Provided one does not get dizzy in logical matters, one may also consult the voluminous article on judgment in the third volume of the fourth edition of Eisler's *Wörterbuch der philosophischen Begriffe.* Whoever has been able to read this article and survives may without gainsay confess with the poet: "To be as-

tonished—that's my lot." Who would think it possible were it not true beyond the shadow of a doubt that even a logician and thinker of the caliber of Lotze interpreted the copula in "S is P" fundamentally as an expression of the relation of identity between S and P? This is very clearly stated in *Grundzüge der Logik* [6] (1922), paragraph 27 which begins with these words: "Stated briefly, the principle of identity asserts: All categorical judgments of the form 'S is P' are false and inadmissible." The reason? The principle of identity asserts expressly: S is S, and *not* P.

There remains, indeed, nothing for us to do than to say that Hermann Weyl, in his masterly *Philosophie der Mathematik* (1926, in Baeumler-Schröter's *Handbuch der Philosophie*) is, by virtue of overwhelming evidence, justified in stating by way of explanation on page 39: "The grotesque examples of confusion of the copula with existence and identity constitute one of the saddest examples of the dependence of philosophical speculation on chance forms of speech."

205. Concerning the status of analysis of existential statements in qualitative logic we learn the frightening facts from Anton Marty: *Gesammelte Schriften,* II, 1 (1918), p. 162-217.

206. How considerable this accomplishment is, is very beautifully confirmed by the Herbartian M. W. Drobisch who declares in his *Logik* [4] (1875), p. 5, that the strictly synthetic development of logic according to mathematical patterns, however desirable it may be in itself, is practically incapable of being carried out. His reason is that thinking is not such a translucent and well-known fact as are mathematical

equations. Let us add, Drobisch *did* know his mathematics.

207. See above, p. 35-6.

208. The best and simplest presentation of this "logic by intension" within symbolic logic is, at present, Hilbert-Ackermann: *Grundzüge der theoretischen Logik* (1928; *Grundlehren der mathematischen Wissenschaften,* vol. XXXVII).

209. *Cf.* H. Rickert: *Die Logik des Prädikats und das Problem der Ontologie* (*SHA,* phil.-hist. Kl., 1930/31, 1. Abh.), p. 49, note: "The idea . . . that several *different* types of logic should be possible is from the very start a logical absurdity. It must be said again and again, in philosophy there seem to be not only immortal concepts and problems, but also immortal instances of—thoughtlessness. Logical relativism is such a 'thought-lessness' in the real sense of the word which would only then be 'consistent' if, on its part, it would *abstain* from all thought. As soon as one *reflects* on several kinds of logic one must, as with every process of thought, use logic and presuppose that only this one logic is valid. Otherwise, reflecting on different 'logics' makes no sense."

In this characteristic proof Rickert has presupposed precisely what is under discussion: that there is only One logic. What would occur if certain syllogistic rules were singled out and eliminated from use, cannot be foreseen at all *a priori.* This can only be accomplished by individual research, painstakingly exact and subject to rigorous control. It has become evident today that a great deal can result from such labor in so far as reductions are concerned. That one must study these endeavors and not prejudge them

is something they share of necessity with *every* undertaking characterized by meticulousness.

210. *Cf.* Bertrand Russell: *Introduction to Mathematical Philosophy* (London, 1919), chapter 16: "Descriptions."

211. *Studien,* III.

212. Included in the *Lehrbuch der Philosophie* edited by Max Dessoir and published by Ullstein.

213. Leisegang furnishes on pages 177-198 a splendid commentary on a part of Hegel's *"Logik,"* the best explanation in general I am acquainted with.

214. P. 438 ff.

215. It comprises 947 pages in large quarto.

216. *WH,* XXVI.

217. Reprinted at Leipzig, 1929-31, at Felix Meiner's.

218. For example, the book in question is not at all reliable so far as the logical use of geometrical and letter symbols is concerned; see p. 228 f.

219. *BT,* 1923.

220. *Elementa Logices Aristoteleae,* 1836 (ninth edition, 1892); *Erläuterungen zu den Elementen der Aristotelischen Logik,* 1842 (third edition, 1876).

221. See above, p. 38.

222. *CAG,* II, 1.

223. *CAG,* II, 2.

224. Second edition, Frankfurt, 1617.

225. Still very much worth reading!

226. The "famous" author of the *Logik als Wissenschaft vom reinen Begriff* (translated from the fourth edition by Felix Noeggerath, 1930). We should go on record to state that the latest logic in the spirit of Hegel *does not satisfy* the demands of rigorous

thinking *at all* while *doing everything possible* to criticize symbolic logic. At the same time it is completely misunderstood (p. 94 f.). Unfortunately, there are also no bounds to the arrogance with which this unchecked "high-level" thinking is pursued.

227. See above, p. 58 and note 208.

228. *Schriften zur wissenschaftlichen Weltauffassung,* ed. by Philipp Frank and Moritz Schlick, vol. 2.

229. *Mathematisch-physikalische Bibliothek,* ed. by W. Lietzmann and A. Witting, No. 71.

230. *Travaux de la Société des Sciences et des Lettres de Varsovie, Classe* III: *Sciences mathématiques et physiques,* No. 33.

231. See above, note 68.

232. Second edition, 1931, published by the Drei-Masken-Verlag, München.

233. *Revue Néo-Scolastique de Philosophie* (Louvain), vol. 26 (1924) and vol. 27 (1925).

234. *Ibid.,* vol. 29 (1927) and vol. 30 (1928).

235. *Cornell Studies in Philosophy,* No. 15.

236. *PM,* I, Introduction to the second edition.

237. *WH,* VII.

238. See above, p. 36.

239. *Op. cit.,* p. 158b.

240. For Aristotle and Aristotelianism it should read: the principle of the excluded middle. Nowhere in Aristotle is there a reference to a limitation of the principle of the excluded contradiction. It is the *noli me tangere* of Aristotelian logic and as such it also passed over into Brouwer's logic. I do not know any logic whatever which, especially in certain statements concerning the future, calls for a limitation of the principle of excluded contradiction. Hegelian

logic demands its nullification for *all* speculative thought in which case it is left to the reader of this logic to fathom what is meant by speculative thought. Moreover, there are some Christian theologians who have set aside the principle of excluded contradiction in favor of the potentialities of divine omnipotence (see above, p. 39, on Peter Damiani). Among Protestants this critique has come to life again in so-called dialectic theology which is oriented on Kierkegaard, but in a way which in my opinion no one has yet been able to explain so that one could follow intelligently.

241. In the problem under consideration for which the nonvalidity of the principle of the excluded middle is maintained we are, so far as Aristotle and Aristotelianism are concerned, dealing only with such statements about future events whose occurrence or nonoccurrence depend on an act of the human will.

242. So far as I understand Lukasiewicz, he does not think at all that the Aristotelian argument is cogent, but only barely discutable. This is sufficient for a starting point of his logic which I think is highly interesting. His logic is the first example, so far as I am aware, of a non-Aristotelian logic in the strict sense, that is, a logic in which statements appear which would be wrong in an Aristotelian logic. This does *not* hold true for Brouwer's logic, for his logic does *not* contradict Aristotelian logic *at all*. Proof: Brouwer does not at all maintain the falsity, but only the *meaninglessness* of the illimited principle of the excluded middle (see above, p. 36). His logic, however, is a very exciting *genuine part* of Aristotelian logic if we look at the consequences. It is nothing

less, but also nothing more. To be sure, we should also mention the radical criticism, reminiscent of Descartes (see above, note 37), of the reduction of the syllogism to the correct application of codified syllogistic rules. Nevertheless, this criticism means in practice only reserving degrees of freedom which up to now have not become effective.

243. This characterization of logical propositions which originated with Ludwig Wittgenstein is very indefinite. Much more definite is the characterization of these propositions, again first made by Wittgenstein, as tautologies. These tautologies coincide with our "perfect forms" (see above, p. 3-4) in so far as we may assert universal validity for them.

244. We are constrained to say the following regarding this interpretation of logical propositions:

1) *On it rests the entire Aristotle critique which we have presented here; it does not rest on the novel interpretation of determinism.*

2) Aristotle agrees with this interpretation. His limitation of the principle of the excluded middle is in my opinion to be interpreted suchwise that it is a logical principle only within the limits in which it has *no* reference to future events whose occurrence or nonoccurrence is dependent on an act of the human will.

3) This limitation has been motivated by the Aristotelian concept of truth. For, *true* is, according to Aristotle, a statement when and only when the facts it has formulated either exist or—in so far as they belong to the future—are determined unambiguously in their existence by present conditions in the world. Of course, we can interpret the unambiguous deter-

mination absolutely in Schlick's sense as precalculability.

4) It is possible to criticize this concept of truth. One can ask what is to be understood by facts formulated in a statement. Truly a serious problem! But we must add immediately that even the latest epistemology with symbolic logic orientation has not found to date an answer which is only near satisfactory. One could go on asking whether it is *practical* to expand this concept of truth to include statements concerning future events in exactly the manner in which Aristotle has done it.

5) Indeed, we cannot see why the truth or falsity of a statement about the occurrence or nonoccurrence of an event E which belongs to the future must be interpreted in such a manner that they imply their determination in the sense of a precalculability. Rather, in order to obtain a reasonable concept of truth or falsity the requirement should be sufficient that such an Aristotelian statement be called *true* or *false* when and only when the occurrence or nonoccurrence of E at the set time and in the designated place can be noted without ambiguity. That of both cases in a situation capable of being checked at all, one and only one has to occur, is, of course, also the opinion of Aristotle. If I see correctly, the Aristotelian limitation of the principle of the excluded middle becomes, thus, superfluous.

6) What have we accomplished, then? Has Aristotle been contradicted by this criticism? If so, we should have had to demonstrate that Aristotle committed a logical error. We will hear of nothing of the sort, for what we have shown has been confined to proving the *impracticability* and *correctibility* of

the concept of truth and falsity which he presupposes in this case.

245. Even an Aristotelian can acknowledge that. But one cannot forbid him to formulate his concept of truth in such a way that certain statements with regard to the future are, nonetheless, not unambiguously true or false even today. To say it once again, one can only dispute the practicability of such a formulation.

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